Data Analysis and Probability

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Chapter Project Online
KEYWORD: MA7 ChProj

Over 170,000 people attended the Austin City Limits Music Festival where more than 130 bands performed.

ARE YOU READY?

✓ Vocabulary

Match each term on the left with a definition on the right.

1. difference
2. factor
3. natural numbers
4. ratio
5. sum

A. the result of an addition

B. a number that is multiplied by another number to get a product

C. numbers that can be expressed in the form $\frac{a}{b}$, where a and b are both integers and $b \neq 0$

D. the result of a subtraction

E. a comparison of two quantities by division

F. the counting numbers: 1, 2, 3, …

✓ Solve Proportions

Solve each proportion.

6. $\frac{3}{4} = \frac{x}{12}$

7. $\frac{15}{9} = \frac{3}{x}$

8. $\frac{10}{20} = \frac{x}{100}$

9. $\frac{250}{1500} = \frac{x}{100}$

✓ Compare and Order Real Numbers

Compare. Write <, >, or =.

10. $20 \ \blacksquare \ 13$

11. $\frac{2}{3} \ \blacksquare \ \frac{1}{2}$

12. $\frac{3}{4} \ \blacksquare \ \frac{7}{9}$

13. $0.75 \ \blacksquare \ \frac{9}{12}$

Order the numbers from least to greatest.

14. $\frac{1}{2}, \frac{4}{5}, \frac{1}{8}, \frac{3}{4}, \frac{2}{3}$

15. $0.12, \frac{2}{5}, \frac{3}{4}, 0.3, \frac{1}{3}$

✓ Multiply Decimals

Multiply.

16. 0.25×300

17. 0.5×4000

18. 0.05×200

19. 0.125×9600

✓ Divide Decimals

Divide.

20. $435 \div 10$

21. $32 \div 100$

22. $777 \div 1000$

23. $295 \div 10{,}000$

✓ Fractions, Decimals, and Percents

Write the equivalent decimal.

24. $\frac{3}{5}$

25. 45%

26. $\frac{3}{4}$

27. 8%

Write the equivalent percent.

28. $\frac{1}{4}$

29. 0.2

30. 0.36

31. $\frac{1}{10}$

Key Vocabulary/Vocabulario

combination	combinación
compound event	suceso compuesto
dependent events	sucesos dependientes
experimental probability	probabilidad experimental
frequency	frecuencia
independent events	sucesos independientes
median	mediana
outlier	valor extremo
permutation	permutación
probability	probabilidad
quartile	cuartil
theoretical probability	probabilidad teórica

Vocabulary Connections

To become familiar with some of the vocabulary terms in the chapter, consider the following. You may refer to the chapter, the glossary, or a dictionary if you like.

1. The *median strip* is the middle region that divides a highway in half. Use this knowledge to define **median** as it relates to a set of data.

2. The word **quartile** starts with the prefix *quart-*. What are some other words that start with the prefix *quart-*? What do they all have in common?

3. **Probability** is the chance something will happen. Based on your understanding of the words *experiment* and *theory*, compare and contrast the terms **experimental probability** and **theoretical probability**.

4. A *compound word* is made up of two or more words. What do you think makes up a **compound event**?

Algebra I TEKS	Les. 10-1	Les. 10-2	Les. 10-3	10-3 Tech. Lab	Les. 10-4	10-5 Alg. Lab	Les. 10-5	10-5 Tech. Lab	Les. 10-6	Les. 10-7	10-7 Alg. Lab	Les. 10-8
A.1.D Foundations for functions* represent relationships among quantities using… tables, graphs…	★	★	★		★							
A.2.D Foundations for functions*… collect and organize data…	★	★	★					★				

* Knowledge and skills are written out completely on pages TX28–TX35.

Reading and Writing Math

Reading Strategy: Read and Interpret Graphics

Knowing how to interpret figures, diagrams, charts, and graphs will help you gather the information you need to get the correct answer.
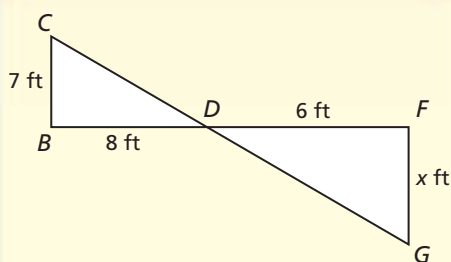
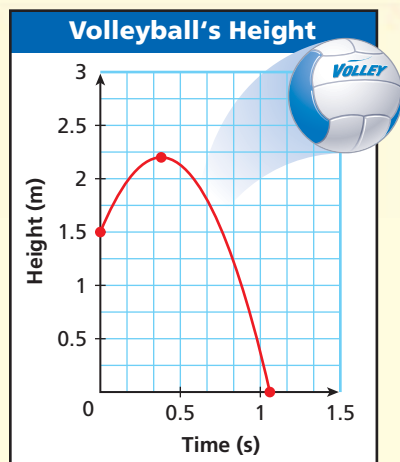| What You See	How to Interpret
	✔ **Read all labels.** $CB = 7$ ft; $BD = 8$ ft; $DF = 6$ ft; $FG = x$ ft ✘ **Do not assume anything.** It appears that $\overline{DF}$ is longer than $\overline{BD}$, but the labels indicate that it is shorter. $\angle B$ and $\angle F$ appear to be 90°, but the diagram does not indicate this.

What You See	How to Interpret
	✔ **Read the title.** "Volleyball's Height" ✔ **Read each axis label.** **x-axis** Indicates **time** measured in seconds **y-axis** Indicates **height** measured in meters ✔ **Determine what information is represented.** Each point on the graph gives the **height** of the ball at that **time**.

Try This

Look up each exercise in the text and answer the corresponding questions.

1. Lesson 2-6 Exercise 42: What is the title of the table? What is the record for the 200-meter run?

2. Lesson 7-7 Exercise 69: What does x represent in the diagram? Make a list of the information you know from the diagram.

3. Lesson 8-3 Exercise 51: What is represented by the y-axis? What is the y-value when x equals 3.5? Is the graph linear? How do you know?

4. Lesson 9-8 Exercise 33: What does the dashed line represent? What facts about the parallelogram are given from the diagram?

Bar and Circle Graphs

Data Analysis

Data displayed in bar graphs and circle graphs can be used to solve equations. In these problems, parts of the graphs are missing.

Example 1

The top part of this graph was torn off. If Warren received 15% of the votes, how many votes did Adams receive?

Step 1 Find the total number of votes. Let t represent the total.

15%	of	the total votes	is	42 votes.
0.15	•	t	=	42

$$0.15t = 42$$
$$t = 280 \text{ votes}$$

**Step 2** Find the number of votes Adams received.

Let a represent the number of votes received by Adams. Let h, w, s, and m represent the number of votes received by Hansen, Warren, Sweeney, and Marino.

$$t = a + h + w + s + m$$
$$280 = a + 52 + 42 + 65 + 28$$
$$280 = a + 187 \qquad \text{\textit{Substitute the numbers shown on the graph.}}$$
$$\underline{-187 \qquad -187} \qquad \text{\textit{Simplify the right side of the equation.}}$$
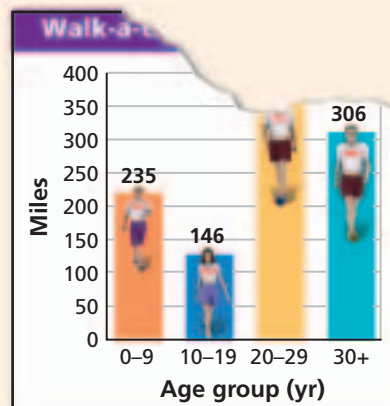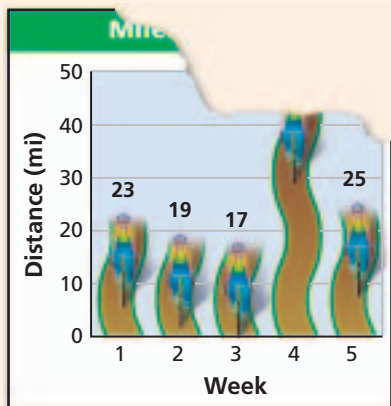$$93 = a \qquad \text{\textit{Subtract 187 from both sides.}}$$

Adams received 93 votes.
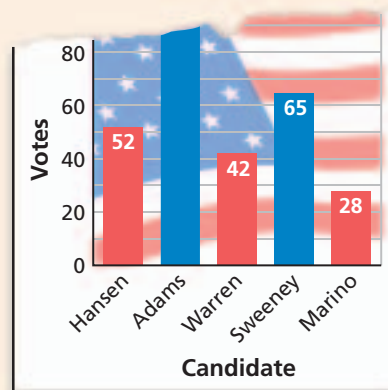
Try This

TAKS | Grade 8 Obj. 5
Grades 9–11 Obj. 9

1. The missing bar is twice as tall as the bar for week 2. How many total miles did Kim bike in these five weeks?

2. People aged 20–29 years walked 275 more miles than the oldest age group. Find the total miles walked by all age groups.

Remember that a circle graph represents all the data in a data set. The percent represented by each section is a part of the whole data set, so the sum of all the percents must be 100%.

Example 2

A survey asked people in a neighborhood to agree or disagree with the following statement:

"We need a traffic light at Jefferson Avenue and Third Street."

If 35% of the people disagreed with the statement, how many people had no opinion?

The number of people who answered "no opinion" is missing from the graph.

Opinion Survey

Strongly agree 36
Strongly disagree 45
Agree 27
Disagree 63
No opinion ?

Step 1 Find the total number of people who answered the survey. Let t represent the total number of people.

35%	of	the total number of people	is	63 people.
0.35	•	t	=	63

$$0.35t = 63$$
$$t = 180 \text{ people}$$

Step 2 Find the number of people who answered "no opinion."

Let n represent the number of "no opinion" answers. Let d, s, g, and a represent the number of "disagree," "strongly disagree," "strongly agree," and "agree" answers.

$$t = n + d + s + g + a$$
$$180 = n + 63 + 45 + 36 + 27 \qquad \text{\textit{Substitute the numbers shown on the graph.}}$$
$$180 = n + 171 \qquad \text{\textit{Simplify the right side of the equation.}}$$
$$\underline{-171 \qquad -171} \qquad \text{\textit{Subtract 171 from both sides.}}$$
$$9 = n$$

There were 9 people who had no opinion.
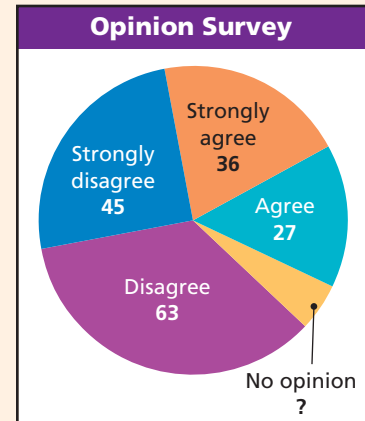
Try This

TAKS | Grade 8 Obj. 5
Grades 9–11 Obj. 9

3. The students in a junior high school voted on their choice for a field trip. Sixteen students voted for the natural history museum. How many students voted for the winning choice?

Field Trip Vote

Aquarium 25%
Zoo 20%
Planetarium ?
Museum 20%

4. At the fall dance recital, 40% of the tickets were sold to adults. What percent of the sales were to seniors?
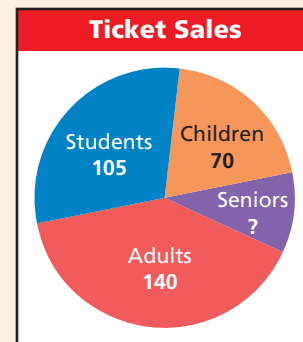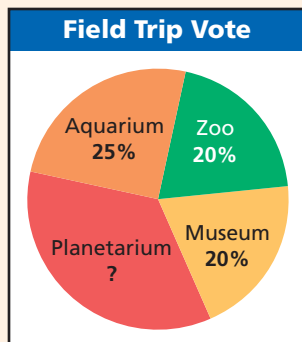
Ticket Sales

Students 105
Children 70
Seniors ?
Adults 140

Organizing and Displaying Data

Objectives
Organize data in tables and graphs.

Choose a table or graph to display data.

Vocabulary
bar graph
line graph
circle graph

Who uses this?
Nutritionists can display health information about food in bar graphs.

Bar graphs, line graphs, and *circle graphs* can be used to present data in a visual way.
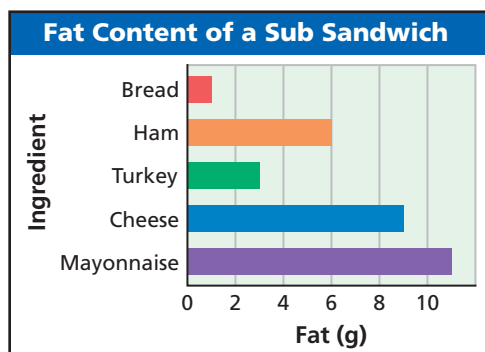
A **bar graph** displays data with vertical or horizontal bars. Bar graphs are a good way to display data that can be organized into categories. Using a bar graph, you can quickly compare the categories.

E X A M P L E   1   Reading and Interpreting Bar Graphs

Use the graph to answer each question.

Fat Content of a Sub Sandwich

A   Which ingredient contains the most fat?
mayonnaise   *The bar for mayonnaise is the longest.*

B   How many more grams of fat are in ham than in turkey?
$6 - 3 = 3$ *There are 6 grams of fat in ham and 3 grams of fat in turkey.*

C   How many total fat grams are in this sandwich?
$1 + 6 + 3 + 9 + 11 = 30$ *Add the number of fat grams for each ingredient.*

D   What percent of the total fat grams in this sandwich are from turkey?
$\dfrac{3}{30} = \dfrac{1}{10} = 10\%$ *Out of 30 total fat grams, 3 fat grams are from turkey.*

CHECK IT OUT!   Use the graph to answer each question.
1a. Which ingredient contains the least amount of fat?
1b. Which ingredients contain at least 8 grams of fat?

A double-bar graph can be used to compare two data sets. A double-bar graph has a key to distinguish between the two sets of data.

EXAMPLE 2 **Reading and Interpreting Double Bar Graphs**

Use the graph to answer each question.

A In which year did State College have the greatest average attendance for basketball?
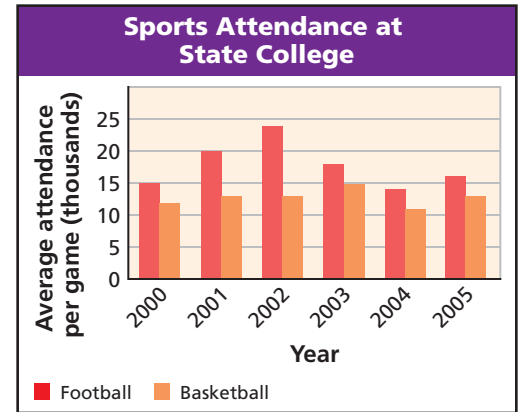
2003

Find the tallest orange bar.

B On average, how many more people attended a football game than a basketball game in 2001?

$20{,}000 - 13{,}000 = 7000$

Find the height of each bar for 2001 and subtract.

Sports Attendance at State College

Average attendance per game (thousands)

■ Football   ■ Basketball

CHECK IT OUT!

2. Use the graph to determine which years had the same average basketball attendance. What was the average attendance for those years?

A **line graph** displays data using line segments. Line graphs are a good way to display data that changes over a period of time.

EXAMPLE 3 **Reading and Interpreting Line Graphs**

Use the graph to answer each question.

Temperature on One Day

Temperature (°F)

Midnight 4:00 A.M. 8:00 A.M. Noon 4:00 P.M. 8:00 P.M.
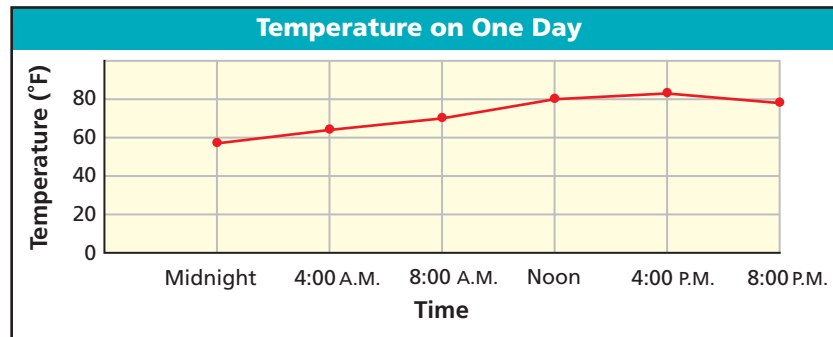
Time

A At what time was the temperature the warmest?

4:00 P.M.   *Identify the highest point.*

B During which 4-hour time period did the temperature increase the most?

From 8:00 A.M. to noon   *Look for the segment with the greatest positive slope.*

CHECK IT OUT!

3. Use the graph to estimate the difference in temperature between 4:00 A.M. and noon.

A double-line graph can be used to compare how two related data sets change over time. A double-line graph has a key to distinguish between the two sets of data.

EXAMPLE **4** **Reading and Interpreting Double-Line Graphs**

Use the graph to answer each question.

Airfare Between Two Cities
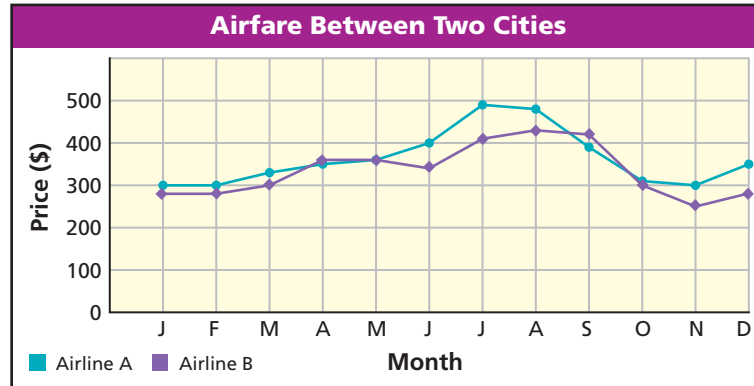
Airline A ■ Airline B ■

A **In which month(s) did airline B charge more than airline A?**
April and September   *Identify the points when the purple line is higher than the blue line.*

B **During which month(s) did the airlines charge the same airfare?**
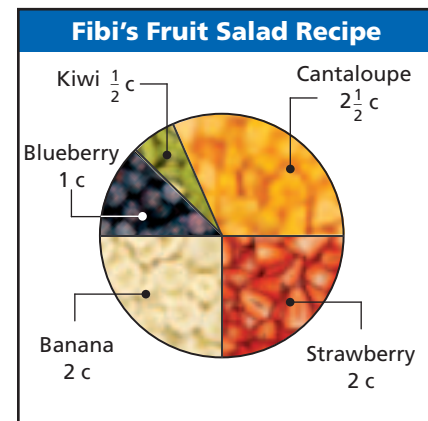May   *Look for the point where the data points overlap.*

4. Use the graph to describe the general trend of the data.

A **circle graph** shows parts of a whole. The entire circle represents 100% of the data and each sector represents a percent of the total. Circle graphs are good for comparing each category of data to the whole set.

EXAMPLE **5** **Reading and Interpreting Circle Graphs**

Use the graph to answer each question.

A **Which two fruits together make up half of the fruit salad?**
bananas and strawberries

Look for two fruits that together make up half of the circle.

Reading Math

The sections of a circle graph are called sectors.

B **Which fruit is used more than any other?**
cantaloupe

Look for the largest sector of the graph.

Fibi's Fruit Salad Recipe

Kiwi $\frac{1}{2}$ c
Cantaloupe $2\frac{1}{2}$ c
Blueberry 1 c
Banana 2 c
Strawberry 2 c

5. Use the graph to determine what percent of the fruit salad is cantaloupe.

EXAMPLE 6

Choosing and Creating an Appropriate Display

Use the given data to make a graph. Explain why you chose that type of graph.

A

Livestock Show Entries	
Animal	**Number**
Chicken	38
Goat	10
Horse	32
Pig	12
Sheep	25

A bar graph is appropriate for this data because it will be a good way to compare categories.

Step 1 Determine an appropriate scale and interval. The scale must include all of the data values. The scale is separated into equal parts, called intervals.

Step 2 Use the data to determine the lengths of the bars. Draw bars of equal width. The bars should not touch.

Step 3 Title the graph and label the horizontal and vertical scales.

Livestock Show Entries
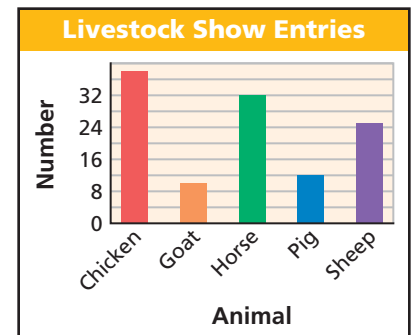
B

Division of Crops	
Crop	**Area (acres)**
Corn	70
Fallow	50
Mixed vegetables	10
Soybeans	40
Wheat	30

A circle graph is appropriate for this data because it shows categories as parts of a whole.

Step 1 Calculate the percent of the total represented by each category.

Corn: $\frac{70}{200} = 0.35 = 35\%$

Soybeans: $\frac{40}{200} = 0.2 = 20\%$

Fallow: $\frac{50}{200} = 0.25 = 25\%$

Wheat: $\frac{30}{200} = 0.15 = 15\%$

Mixed vegetables: $\frac{10}{200} = 0.05 = 5\%$

Step 2 Find the angle measure for each sector of the graph. Since there are 360° in a circle, multiply each percent by 360°.

Corn: $0.35 \times 360° = 126°$
Fallow: $0.25 \times 360° = 90°$
Mixed vegetables: $0.05 \times 360° = 18°$
Soybeans: $0.2 \times 360° = 72°$
Wheat: $0.15 \times 360° = 54°$

Step 3 Use a compass to draw a circle. Mark the center and use a straightedge to draw one radius. Then use a protractor to draw each central angle.

Step 4 Title the graph and label each sector.

Division of Crops

TEXAS LINK

Food

The Texas Ruby Red grapefruit was the first citrus fruit to be granted a U.S. Patent.
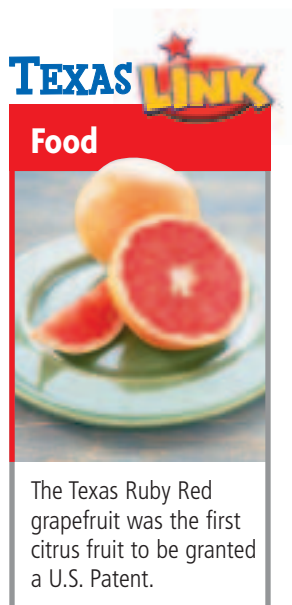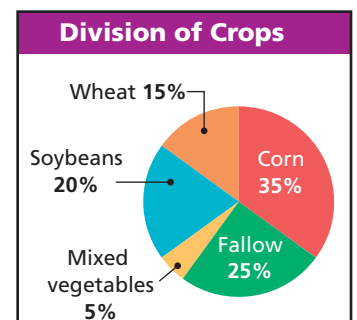
Use the given data to make a graph. Explain why you chose that type of graph.

C

Chinnick College Enrollment	
Year	Students
1930	586
1955	2,361
1980	15,897
2005	21,650

A line graph is appropriate for this data because it will show the change in enrollment over a period of time.

Step 1 Determine the scale and interval for each set of data. Time should be plotted on the horizontal axis because it is independent.

Step 2 Plot a point for each pair of values. Connect the points using line segments.

Step 3 Title the graph and label the horizontal and vertical scales.

CHECK IT OUT!
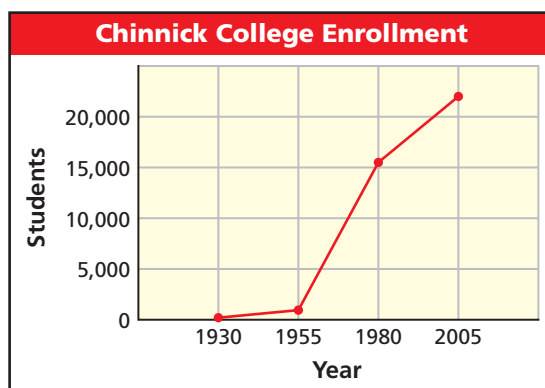
6. Use the given data to make a graph. Explain why you chose that type of graph.

The data below shows how Vera spends her time during a typical 5-day week during the school year.

Vera's Schedule						
Activity	Sleeping	Eating	School	Sports	Homework	Other
Time (h)	45	8	30	10	10	17

THINK AND DISCUSS

1. What are some comparisons you can make by looking at a bar graph?

2. Name some key components of a good line graph.

3. GET ORGANIZED Copy and complete the graphic organizer. In each box, tell which kind of graph is described.

Graph Type
- Compares categories
- Shows change over time
- Shows how a whole is divided in parts
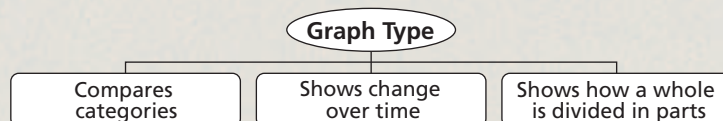
GUIDED PRACTICE

Vocabulary Use the vocabulary from this lesson to answer the following questions.

1. In a *circle graph*, what does each sector represent?

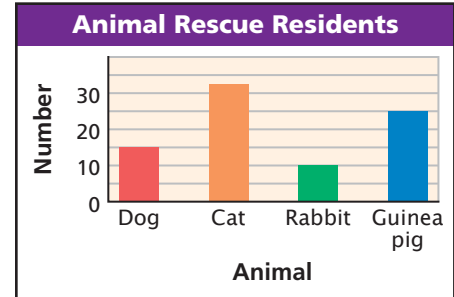2. In a *line graph*, how does the slope of a line segment relate to the rate of change?

SEE EXAMPLE 1
p. 678

Use the bar graph for Exercises 3 and 4.

3. Estimate the total number of animals at the shelter.

4. There are 3 times as many ___?___ as ___?___ at the animal shelter.
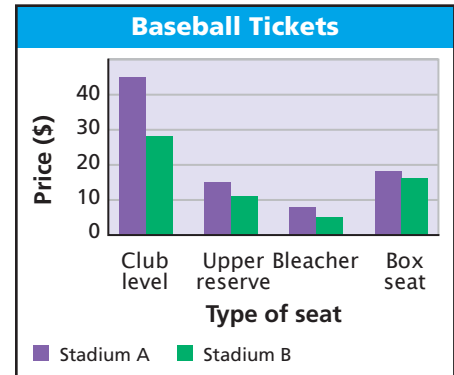
Animal Rescue Residents

SEE EXAMPLE 2
p. 679

Use the double-bar graph for Exercises 5–7.

5. About how much more is a club level seat at stadium A than at stadium B?

6. Which type of seat is the closest in price at the two stadiums?

7. Describe one relationship between the ticket prices at stadium A and stadium B.

Baseball Tickets

SEE EXAMPLE 3
p. 679

Use the line graph for Exercises 8 and 9.

8. Estimate the number of tickets sold during the week of the greatest sales.

9. Which one-week period of time saw the greatest change in sales?

Movie Ticket Sales
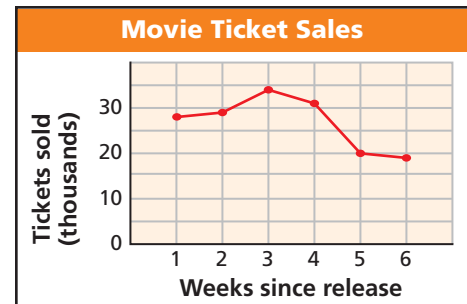
SEE EXAMPLE 4
p. 680

Use the double-line graph for Exercises 10–12.

10. When was the support for the two candidates closest?

11. Estimate the difference in voter support for the two candidates five weeks before the election.

12. Describe the general trend(s) of voter support for the two candidates.

Election Poll

10-1 Organizing and Displaying Data **683**

Use the circle graph for Exercises 13–15.

13. Which color is least represented in the ball playpen?

14. There are 500 balls in the playpen. How many are yellow?

15. Which two colors are approximately equally represented in the ball playpen?

Colors in Ball Playpen

Yellow 24%
Blue 18%
Green 17%
Red 31%
Purple 10%

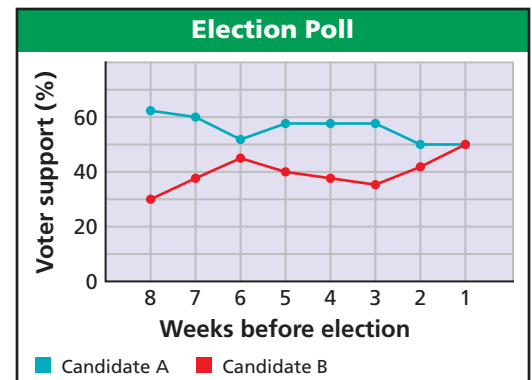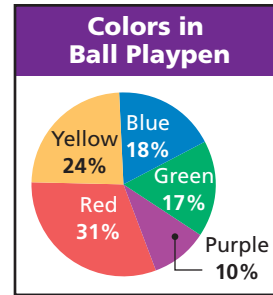**16.** The table shows the breakdown of Karim's monthly budget of $100. Use the given data to make a graph. Explain why you chose that type of graph.

Item/Activity	Spending ($)
Clothing	35
Food	25
Entertainment	25
Other	15

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
17–18	1
19–21	2
22–23	3
24–26	4
27–28	5
29	6

TEKS ★ TAKS

Use the bar graph for Exercises 17 and 18.

17. Estimate the difference in population between the tribes with the largest and the smallest population.

18. Approximately what percent of the total population shown in the table is Cherokee?

Native American Population, 2000

Tribe: Cherokee, Chippewa, Choctaw, Navajo, Sioux
Population (thousands): 0 50 100 150 200 250 300
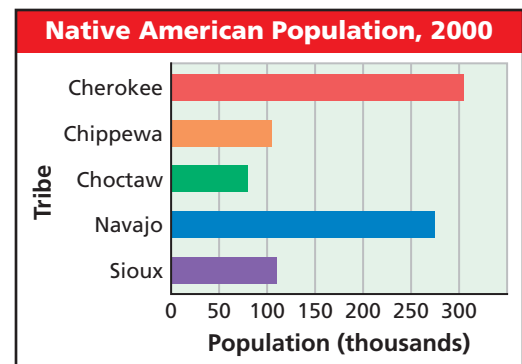
Source: U.S. Census Bureau

Use the double bar graph for Exercises 19–21.

19. On what day did Ray do the most overall business?

20. On what day did Ray have the busiest lunch?

21. On Sunday, about how many times as great was the number of dinner customers as the number of lunch customers?

Ray's Restaurant

Customers: 0 50 100 150 200 250
Day of the week: S M T W T F S
■ Lunch ■ Dinner
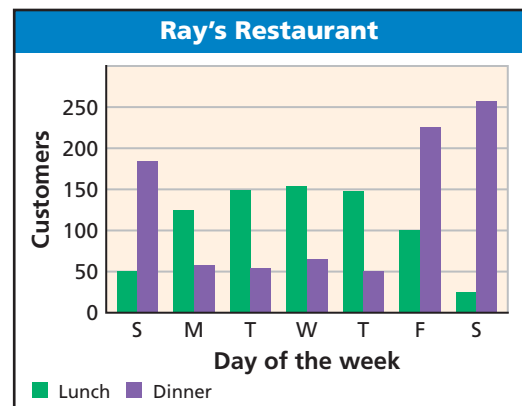
Use the line graph for Exercises 22 and 23.

22. Between which two games did Marlon's score increase the most?

23. Between which three games did Marlon's score increase by about the same amount?

Marlon's Video Game Scores

Score: 0 400 800 1200
Game number: 1 2 3 4 5 6

Use the double-line graph for Exercises 24–26.

24. What was the average value per share of Juan's two stocks in July 2004?

25. Which stock's value changed the most over any time period?

26. Describe the trend of the values of both stocks.

Juan's Stock Values

Use the circle graph for Exercises 27 and 28.

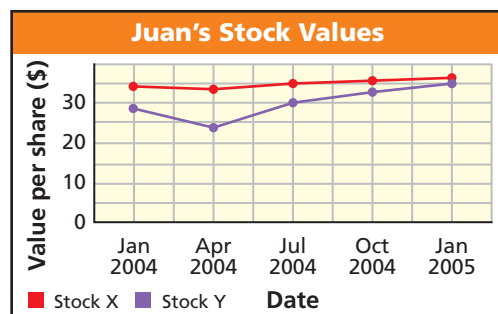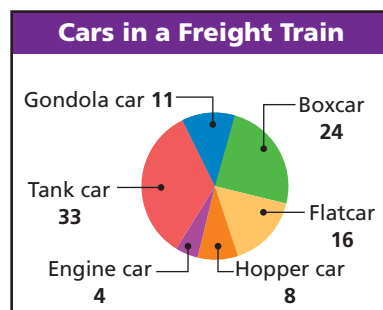**27.** About what percent of the total number of cars are hopper cars?

28. About what percent of the total number of cars are gondola or tank cars?

Cars in a Freight Train

29. The table shows the weight of twin babies at various times from birth to four weeks old. Use the given data to make a graph. Explain why you chose that type of graph.

Age (days)	Boy's Weight (lb)	Girl's Weight (lb)
1	5.3	5.7
3	5.0	5.2
7	5.5	5.9
14	6.2	6.8
28	7.9	7.5

Write *bar, double-bar, line, double-line,* or *circle* to indicate the type of graph that would best display the data described.

30. attendance at a carnival each year over a ten-year period

31. attendance at two different carnivals each year over a ten-year period

32. attendance at five different carnivals during the same year

33. attendance at a carnival by age group as it relates to total attendance

34. Critical Thinking Give an example of real-world data that would best be displayed by each type of graph: line graph, circle graph, double-bar graph.

MULTI-STEP TAKS PREP

35. This problem will prepare you for the Multi-Step TAKS Prep on page 710.

The first modern Olympic Games took place in 1896 in Athens, Greece. The circle graph shows the total number of medals won by several countries at the Olympic Games of 1896.

a. Which country won the most medals? Estimate the percent of the medals won by this country.

b. Which country won the second most medals? Estimate the percent of the medals won by this country.

Olympic Medals, 1896
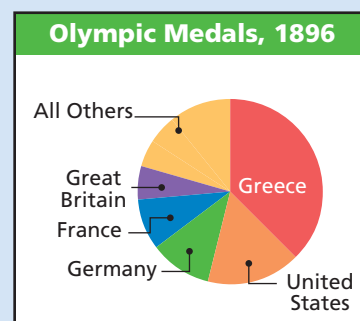
36. Write About It Explain how you could use a line graph to make predictions.

37. Which type of graph would best display the contribution of each high school basketball player to the team, in terms of points scored?

Ⓐ Bar graph   Ⓑ Line graph   Ⓒ Double-line graph   Ⓓ Circle graph

38. At what age did Marianna have 75% more magazine subscriptions than she did at age 40?

Ⓕ 25
Ⓖ 30
Ⓗ 35
Ⓙ 45

Marianna's Magazines

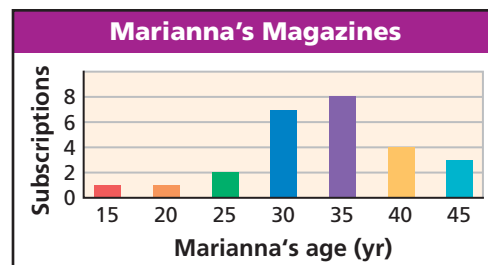**39. Short Response** The table shows the number of students in each algebra class. Make a graph to display the data. Explain why you chose that type of graph.

Teacher	Students
Mr. Abrams	34
Ms. Belle	29
Mr. Marvin	25
Ms. Swanson	27

CHALLENGE AND EXTEND

Students and teachers at Lauren's school went on one of three field trips.

40. On which trip were there more boys than girls?

41. A total of 60 people went to the museum. Estimate the number of girls who went to the museum.

42. Explain why it is not possible to determine whether fewer teachers went to the museum than to the zoo or the opera.

Field Trip Attendance
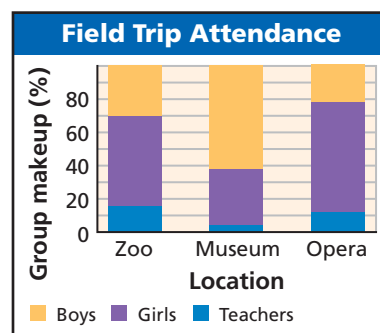
SPIRAL REVIEW

Find the domain and range for each relation and tell whether the relation is a function. *(Lesson 4-2)*

43. $\{(-3, 3), (-1, 1), (0, 0), (1, 1), (3, 3)\}$

44.

x	1	2	3	4	5
y	2	4	6	8	10

45. Triangle ABC has vertices on a coordinate plane as follows:

$A = (0, 5)$, $B = (3, 0)$, $C = (8, 3)$. Show that $\triangle ABC$ is a right triangle. *(Lesson 5-8)*

Classify each polynomial according to its degree and number of terms. *(Lesson 7-5)*

46. $24y$

47. $3x^2 + 6$

48. $4m - 18m^2 - 45m^3 + 120$

10-2 Frequency and Histograms

Objectives
Create stem-and-leaf plots.

Create frequency tables and histograms.

Vocabulary
stem-and-leaf plot
frequency
frequency table
histogram
cumulative frequency

Why learn this?
Stem-and-leaf plots can be used to organize data, like the number of students in elective classes. (See Example 1.)

A **stem-and-leaf** plot arranges data by dividing each data value into two parts. This allows you to see each data value.

The digits other than the last digit of each value are called a stem.

2|3

Key: 2|3 means 23

The last digit of a value is called a leaf.

The key tells you how to read each value.

EXAMPLE 1 Making a Stem-and-Leaf Plot

A The numbers of students in each of the elective classes at a school are given below. Use the data to make a stem-and-leaf plot.

24, 14, 12, 25, 32, 18, 23, 24, 9, 18, 34, 28, 24, 27

Number of Students in Elective Classes

Stem	Leaves
0	9
1	2 4 8 8
2	3 4 4 4 5 7 8
3	2 4

Key: 2|3 means 23

The tens digits are the stems.
The ones digits are the leaves. List the leaves from least to greatest within each row.
Title the graph and add a key.

B Marty's and Bill's scores for ten games of bowling are given below. Use the data to make a back-to-back stem-and-leaf plot.

Marty: 137, 149, 167, 134, 121, 127, 143, 123, 168, 162
Bill: 129, 138, 141, 124, 139, 160, 149, 145, 128, 130

Bowling Scores

Marty		Bill
7 3 1	12	4 8 9
7 4	13	0 8 9
9 3	14	1 5 9
	15	
8 7 2	16	0

Key: |14|1 means 141
3|14| means 143

The first two digits are the stems.
The ones digits are the leaves.
Put Marty's scores on the left side and Bill's scores on the right.
Title the graph and add a key.
The graph shows that three of Marty's scores were higher than Bill's highest score.

Writing Math
Stems are always consecutive numbers. In Example 1B, neither player has scores that start with 15, so there are no leaves in that row.

CHECK IT OUT!

1. The temperatures in degrees Celsius for two weeks are given below. Use the data to make a stem-and-leaf plot.

7, 32, 34, 31, 26, 27, 23, 19, 22, 29, 30, 36, 35, 31

The **frequency** of a data value is the number of times it occurs. A **frequency table** shows the frequency of each data value. If the data is divided into intervals, the table shows the frequency of each interval.

E X A M P L E   2   Making a Frequency Table

The final scores for each golfer in a tournament are given below. Use the data to make a frequency table with intervals.

77, 71, 70, 82, 75, 76, 72, 70, 77, 74, 71, 75, 68, 72, 75, 74

Step 1 Identify the least and greatest values.

The least value is 68. The greatest value is 82.

Step 2 Divide the data into equal intervals.

For this data set, use an interval of 3.

Step 3 List the intervals in the first column of the table. Count the number of data values in each interval and list the count in the last column. Give the table a title.

Golf Tournament Scores	
Scores	Frequency
68–70	3
71–73	4
74–76	6
77–79	2
80–82	1

2. The numbers of days of Maria's last 15 vacations are listed below. Use the data to make a frequency table with intervals.

4, 8, 6, 7, 5, 4, 10, 6, 7, 14, 12, 8, 10, 15, 12

A **histogram** is a bar graph used to display the frequency of data divided into equal intervals. The bars must be of equal width and should touch, but not overlap.

E X A M P L E   3   Making a Histogram

Use the frequency table in Example 2 to make a histogram.

Step 1 Use the scale and interval from the frequency table.

Step 2 Draw a bar for the number of scores in each interval.

All bars should be the same width. The bars should touch, but not overlap.

Step 3 Title the graph and label the horizontal and vertical scales.

> **Helpful Hint**
>
> The intervals in a histogram must be of equal size.

Golf Tournament Scores
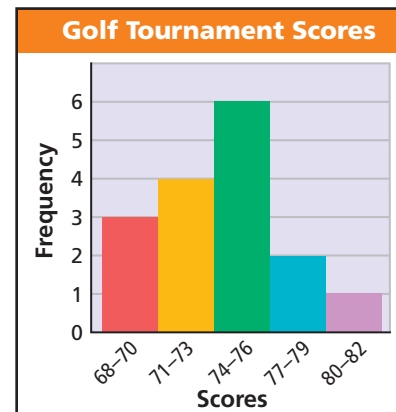
3. Make a histogram for the number of days of Maria's last 15 vacations.

4, 8, 6, 7, 5, 4, 10, 6, 7, 14, 12, 8, 10, 15, 12

Cumulative frequency shows the frequency of all data values less than or equal to a given value. You could just count the number of values, but if the data set has many values, you might lose track. Recording the data in a cumulative frequency table can help you keep track of the data values as you count.

EXAMPLE **4** **Making a Cumulative Frequency Table**

The heights in inches of the players on a school basketball team are given below.

72, 68, 71, 70, 73, 69, 79, 76, 72, 75, 72, 74, 68, 70, 69, 75, 72, 71, 73, 76

a. Use the data to make a cumulative frequency table.

Step 1 Choose intervals for the first column of the table.

Step 2 Record the frequency of values in each interval for the second column.

Step 3 Add the frequency of each interval to the frequencies of all the intervals before it. Put that number in the third column of the table.

Step 4 Title the table.

Basketball Players' Heights		
Height (in.)	Frequency	Cumulative Frequency
68–70	6	6
71–73	8	14
74–76	5	19
77–79	1	20

b. How many players have heights under 74 in?

All heights under 74 in. are displayed in the first two rows of the table, so look at the cumulative frequency shown in the second row.

There are 14 players with heights under 74 in.

CHECK IT OUT!

4. The numbers of vowels in each sentence of a short essay are listed below.

33, 36, 39, 37, 34, 35, 43, 35, 28, 32, 36, 35, 29, 40, 33, 41, 37

a. Use the data to make a cumulative frequency table.

b. How many sentences contain 35 vowels or fewer?

THINK AND DISCUSS

1. In a stem-and-leaf plot, the number of _____?_____ is always the same as the number of data values. (*stems* or *leaves*)

2. Explain how to make a histogram from a stem-and-leaf plot.

3. GET ORGANIZED Copy and complete the graphic organizer.

Know it! Note

Bar Graphs vs Histograms

How are they alike?   How are they different?
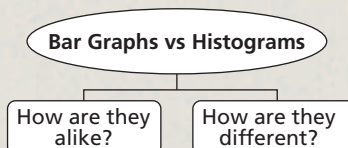
GUIDED PRACTICE

1. **Vocabulary** A(n) _____?_____ is a data display that shows individual data values. (*stem-and-leaf plot* or *histogram*)

SEE EXAMPLE **1**
p. 687

2. **Sports** The ages of professional basketball players at the time the players were recruited are given. Use the data to make a stem-and-leaf plot.

Ages When Recruited
21   23   21   18   22   19   24   22   21   22   20   21

3. **Weather** The average monthly rainfall for two cities (in inches) is given below. Use the data to make a back-to-back stem-and-leaf plot.

Average Monthly Rainfall (in.)											
Austin, TX	1.9	2.4	1.9	3.0	3.6	3.3	1.9	2.1	3.2	3.5	2.2	2.3
New York, NY	3.3	3.1	3.9	3.7	4.2	3.3	4.1	4.1	3.6	3.3	4.2	3.6

SEE EXAMPLE **2**
p. 688

4. **Sports** The finishing times of runners in a 5K race, to the nearest minute, are given. Use the data to make a frequency table with intervals.

Finishing Times in 5K Race (to the nearest minute)
19   25   23   29   32   30   21   22   24
19   28   26   31   34   30   28   25   24

SEE EXAMPLE **3**
p. 688

5. **Biology** The breathing intervals of gray whales are given. Use the frequency table to make a histogram for the data.

Breathing Intervals (min)	
Interval	**Frequency**
5–7	4
8–10	7
11–13	7
14–16	8

SEE EXAMPLE **4**
p. 689

6. The scores made by a group of eleventh-grade students on the mathematics portion of the SAT are given.

Scores on Mathematics Portion of SAT
520   560   720   690   540   630   790   540
600   580   710   500   540   660   630

   a. Use the data to make a cumulative frequency table.

   b. How many students scored 650 or higher on the mathematics portion of the SAT?

PRACTICE AND PROBLEM SOLVING

7. The numbers of people who visited a park each day over two weeks during different seasons are given below. Use the data to make a back-to-back stem-and-leaf plot.

Visitors to a Park													
Summer	25	25	26	27	27	57	59	22	23	29	22	23	54	53
Winter	11	12	13	9	30	27	4	19	14	19	21	33	35	9

8. Weather The daily high temperatures in degrees Fahrenheit in a town during one month are given. Use the data to make a stem-and-leaf plot.

Daily High Temperatures (°F)
68 72 79 77 70 72 75 71 64 64
68 62 70 71 78 83 83 87 91 89
87 75 73 70 69 69 62 58 71 76

9. The overall GPAs of several high school seniors are given. Use the data to make a frequency table with intervals.

Overall GPAs
3.6 2.9 3.1 3.0 2.5 2.6 3.8 2.9
2.2 2.9 3.1 3.3 3.6 3.0 2.3 2.8 2.9

10. Chemistry The atomic masses of the nonmetal elements are given in the table. Use the frequency table to make a histogram for the data.

Atomic Masses of Nonmetal Elements					
Interval	0–49.9	50–99.9	100–149.9	150–199.9	200–249.9
Frequency	11	3	2	0	2

11. The numbers of pretzels found in several samples of snack mix are given in the table.

a. Use the data to make a cumulative frequency table.

b. How many samples of snack mix had fewer than 42 pretzels?

Numbers of Pretzels
42 39 39 38 40
41 44 42 38 44
47 36 40 40 43 38

12. Automobiles The table shows gas mileage for the most economical cars in July 2004, including three hybrids.

Gas Mileage of Economical Cars	
Mileage in City (mi/gal)	32 60 48 38 36 60 35 38 32
Mileage on Highway (mi/gal)	38 51 47 46 47 66 43 46 40

Make a back-to-back stem-and-leaf plot for the data.

13. Damien's math test scores are given in the table:

a. Make a stem-and-leaf plot of Damien's test scores.

b. Make a histogram of the test scores using intervals of 5.

c. Make a histogram of the test scores using intervals of 10.

d. Make a histogram of the test scores using intervals of 20.

e. How does the size of the interval affect the appearance of the histogram?

f. **Write About It** Which histogram makes Damien's grades look highest? Explain.

Damien's Math Test Scores
75 84 68
72 59 88
72 77 81
84 60 70

14. /// ERROR ANALYSIS /// Two students made stem-and-leaf plots for the following data: 530, 545, 550, 555, 570. Which is incorrect? Explain the error.

A

Stem	Leaves
53	0
54	5
55	0 5
57	0

Key: 52|5 means 525

B

Stem	Leaves
53	0
54	5
55	0 5
56	
57	0

Key: 52|5 means 525

Automobiles

Solar cars usually weigh between 330 and 880 pounds. A conventional car weighs over 4000 pounds.

15. This problem will prepare you for the Multi-Step TAKS Prep on page 710.
The 2004 Olympic results for women's weightlifting in the 48 kg weight class are 210, 205, 200, 190, 187.5, 182.5, 180, 177.5, 175, 172.5, 170, 167.5, and 165, measured in kilograms. Medals are awarded to the athletes who can lift the most weight.

  a. Create a frequency table beginning at 160 and using intervals of 10 kg.

  b. Create a histogram of the data.

  c. Tara Cunningham from the United States lifted 172.5 kg. Did she win a medal? How do you know?

16. **Entertainment** The top ten movies in United States theaters for the weekend of June 25–27, 2004, grossed the following amounts (in millions of dollars). Create a histogram for the data. Make the first interval 5–9.9.

Ticket Sales (million $)				
23.9	19.7	18.8	13.5	13.1
11.2	10.2	7.5	6.1	5.1

17. **Critical Thinking** Margo's homework assignment is to make a data display of some data she finds in a newspaper. She found a frequency table with the given intervals.

Explain why Margo must be careful when drawing the bars of the histogram.

Age
Under 18
18–30
31–54
55 and older

18. What data value occurs most often in the stem-and-leaf plot?

  (A) 7

  (B) 4.7

  (C) 47

  (D) 777

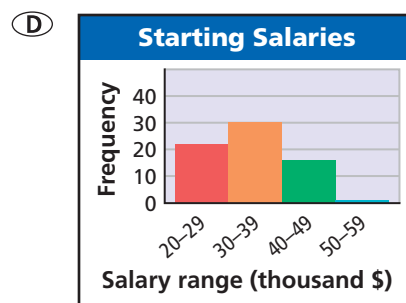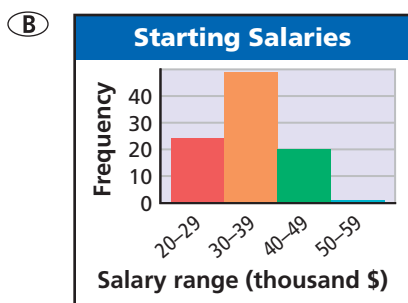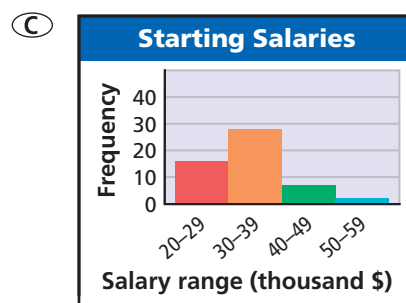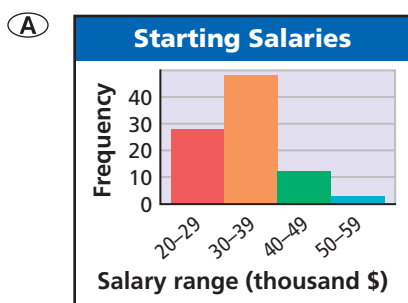| Stem	Leaves
3	2 3 4 4 7 9
4	0 1 5 7 7 7 8
5	1 2 2 3

Key: 3|2 means 3.2

19. The table shows the results of a survey about time spent on the Internet each month. Which statement is NOT supported by the data in the table?

Time Spent on the Internet per Month		
Time (h)	Frequency	Cumulative Frequency
0–4	4	4
5–9	6	10
10–14	3	13
15–19	16	29
20–24	12	41
25–29	7	48
30–34	2	50

  (F) The interval of 30 to 34 h/mo has the lowest frequency.

  (G) More than half of those who responded spend more than 20 h/mo on the Internet.

  (H) Only four people responded that they spend less than 5 h/mo on the Internet.

  (J) Sixteen people responded that they spend less than 20 h/mo on the Internet.

20. The frequencies of starting salary ranges for college graduates are noted in the table. Which histogram best reflects the data?

Starting Salaries	
Salary Range ($)	**Frequency**
20,000–29,000	�HT HT HT HT II
30,000–39,000	HT HT HT HT HT HT
40,000–49,000	HT HT HT I
50,000–59,000	I

Ⓐ
Starting Salaries

Ⓒ
Starting Salaries

Ⓑ
Starting Salaries

Ⓓ
Starting Salaries

CHALLENGE AND EXTEND

21. The cumulative frequencies of each interval have been given. Use this information to complete the frequency column.

Interval	Frequency	Cumulative Frequency
13–16	▦	8
17–20	▦	16
21–24	▦	57
25–28	▦	123

SPIRAL REVIEW

Solve each equation. *(Lessons 2-3 and 2-4)*

**22.** $19 = -2c + 5$

23. $4(m + 2) = -1.2$

24. $2(x - 3) + 7 = 3x - 9$

25. The U.S. standard railroad gauge is 56.5 inches, which is the distance between the track's rails. Charles has a model train whose scale is 113:1. What is the distance between the rails on his model train track? *(Lesson 2-6)*

Use the circle graph for Exercises 26–28. *(Lesson 10-1)*

26. Which two types of gifts make up just over half of the donated gifts?

27. Which type of gift represents $\frac{1}{5}$ of the total donated gifts?

28. If there were 160 gifts donated, how many were books?
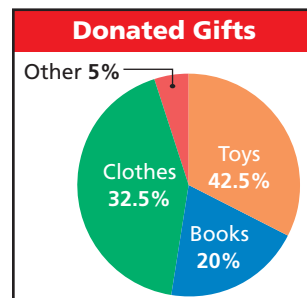
Donated Gifts

Other **5%**
Clothes **32.5%**
Toys **42.5%**
Books **20%**

Data Distributions

Objectives
Describe the central tendency of a data set.

Create box-and-whisker plots.

Vocabulary
mean
median
mode
range
outlier
quartile
interquartile range (IQR)
box-and-whisker plot

Who uses this?

Sports analysts examine data distributions. (See Example 3.)

A *measure of central tendency* describes how data clusters around a value.

- The **mean** is the sum of the values in the set divided by the number of values in the set.

University of Texas softball pitcher Cat Osterman played on the 2004 U.S. Olympic softball team.

- The **median** is the middle value when the values are in numerical order, or the mean of the two middle values if there are an even number of values.

- The **mode** is the value or values that occur most often. There may be one mode or more than one mode. If no value occurs more often than another, we say the data set has no mode.

The **range** of a set of data is the difference between the least and greatest values in the set. The range describes the spread of the data.

E X A M P L E   1

Finding Mean, Median, Mode, and Range of a Data Set

Find the mean, median, mode, and range of each data set.

A The number of hours Isaac did homework on six days: 3, 8, 4, 6, 5, 4.

3, 4, 4, 5, 6, 8   *Write the data in numerical order.*

mean: $\dfrac{3 + 4 + 4 + 5 + 6 + 8}{6} = \dfrac{30}{6}$ *Add all the values and divide by the number of values.*

$= 5$

median: 3, 4, (4, 5,) 6, 8   *There are an even number of values. Find the mean of the two middle values.*
        The median is 4.5.

mode: 4   *4 occurs more often than any other value.*

range: $8 - 3 = 5$

B The weight in pounds of Maria's five cats: 12, 14, 12, 16, 16.

12, 12, 14, 16, 16   *Write the data in numerical order.*

mean: $\dfrac{12 + 12 + 14 + 16 + 16}{5} = \dfrac{70}{5}$ *Add all the values and divide by the number of values.*

$= 14$

median: 12, 12, (14,) 16, 16   *There are an odd number of values. Find the middle value.*
        The median is 14.

mode: 12, 16   *12 and 16 both occur more often than any other value.*

range: $16 - 12 = 4$

CHECK IT OUT!

Find the mean, median, mode, and range of each data set.

1a. 8, 8, 14, 6   **1b.** 1, 5, 7, 2, 3   **1c.** 12, 18, 14, 17, 12, 18

A value that is very different from the other values in the set is called an **outlier**. In the data below, one value is much greater than the other values. This causes the mean to be greater than all of the other data values. In this case, either the median or mode would better describe the data.
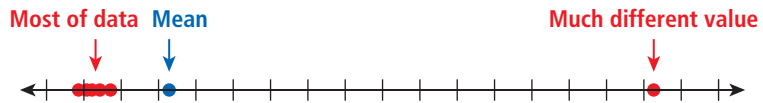
E X A M P L E **2** **Choosing a Measure of Central Tendency**

Niles scored 70, 74, 72, 71, 73 and 96 on six geography tests. Use the mean, median, and mode of his scores to answer each question.

mean = 76   median = 72.5   mode: none

A **Which value gives Niles' test average?**
The average of Niles' scores is the mean, 76.

B **Which value best describes Niles' scores? Explain.**
The median score is the best description of Niles' six scores. Most of his scores were near 72.

The mean is higher than most of Niles' scores because he scored 96 on one test. Since there is no mode, it is not a good description of the data.

2. Josh scored 75, 75, 81, 84, and 85 on five tests. Use the mean, median, and mode of his scores to answer each question.

mean = 80   median = 81   mode = 75

a. Which value describes the score Josh received most often?

b. Which value best describes Josh's scores? Explain.

Measures of central tendency describe how data tends toward one value. You may also need to know how data is spread out across several values.

Helpful Hint

Mathematically, any value that is 1.5(IQR) less than the first quartile or 1.5(IQR) greater than the third quartile is an outlier.

Quartiles divide a data set into four equal parts. Each quartile contains one-fourth of the values in the set. The **interquartile range (IQR)** is the difference between the upper and lower quartiles. The IQR represents the middle half of the data.
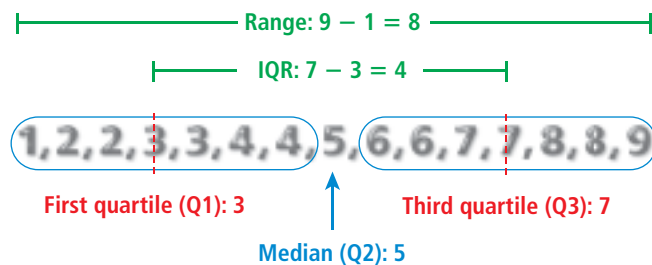
A **box-and-whisker plot** can be used to show how the values in a data set are distributed. The minimum is the least value that is not an outlier. The maximum is the greatest value that is not an outlier. You need five values to make a box-and-whisker plot: the minimum, first quartile, median, third quartile, and maximum.

EXAMPLE **3** *Sports Application*

The numbers of runs scored by a softball team in 19 games are given. Use the data to make a box-and-whisker plot.
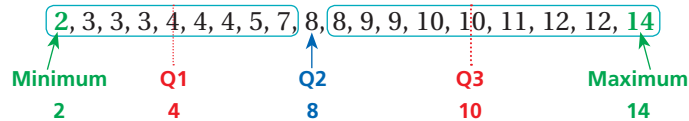
3, 4, 8, 12, 7, 5, 4, 12, 3, 9, 11, 4, 14, 8, 2, 10, 3, 10, 9

Step 1 Order the data from least to greatest.

2, 3, 3, 3, 4, 4, 4, 5, 7, 8, 8, 9, 9, 10, 10, 11, 12, 12, 14

Step 2 Identify the five needed values and determine whether there are any outliers.

2, 3, 3, 3, 4, 4, 4, 5, 7, 8, 8, 9, 9, 10, 10, 11, 12, 12, 14

Minimum	Q1	Q2	Q3	Maximum
2	4	8	10	14

IQR: $10 - 4 = 6 \rightarrow 1.5(6) = 9$

$4 - 9 = -5$ $10 + 9 = 19$

No values are less than -5 or greater than 19, so there are no outliers.

Step 3 Draw a number line and plot a point above each of the five values you just identified. Draw a box through the lower and upper quartiles and a vertical line through the median. Draw lines from the box to the lower and upper extremes. (These are the whiskers.)
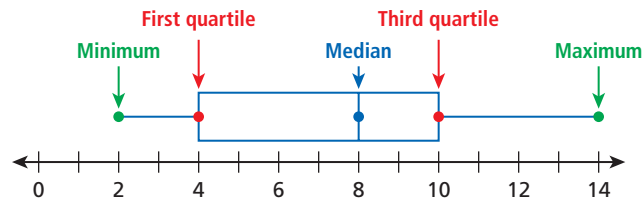
Half of the scores are between 4 and 10 runs per game. One-fourth of the scores are between 2 and 4. The greatest score earned by this team is 14.

3. Use the data to make a box-and-whisker plot.

13, 14, 18, 13, 12, 17, 15, 12, 13, 19, 11, 14, 14, 18, 22, 23

Writing Math

An outlier is represented on a box-and-whisker plot by a point that is not connected to the box by whiskers.

THINK AND DISCUSS

1. Explain when the median is a value in the data set.

2. GET ORGANIZED Copy and complete the graphic organizer. Tell which measure of central tendency answers each question.

Measures of Central Tendency	
Measure	**Used to Answer**
	What is the average?
	What is the halfway point of the data?
	What is the most common value?

10-3 **Exercises**

TAKS | Grade 8 Obj. 5, 6
Grades 9–11 Obj. 3, 9, 10

go.hrw.com/A1/TX
Homework Help Online
KEYWORD: MA7 10-3
Parent Resources Online
KEYWORD: MA7 Parent

GUIDED PRACTICE

1. **Vocabulary** What is the difference between the *range* and the *interquartile range*?

SEE EXAMPLE **1**
p. 694

Find the mean, median, mode, and range of each data set.

2. 85, 83, 85, 82
3. 12, 22, 33, 34, 44, 44
4. 10, 26, 25, 10, 20, 22, 25, 20
5. 71, 73, 75, 78, 78, 80, 85, 86

SEE EXAMPLE **2**
p. 695

6. The distance between five students' homes and the school are 3, 2, 2, 2, and 15 miles. Use the mean, median, and mode of the distances to answer each question.

mean = 4.8   median = 2   mode = 2

   a. Which value describes the distance between home and school that occurs most often?
   b. Which value best describes the distance between home and school? Explain.

SEE EXAMPLE **3**
p. 696

Use the data to make a box-and-whisker plot.

7. 21, 31, 26, 24, 28, 26
8. 12, 13, 42, 62, 62, 82
9. 2, 1, 3, 1, 2, 6, 2, 4
10. 104, 68, 90, 96, 101, 106, 95, 88

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
11–14	1
15	2
16–19	3

TEKS ✦ TAKS

Skills Practice p. S22
Application Practice p. S37

Find the mean, median, mode, and range of each data set.

11. 75, 63, 89, 91
12. 1, 2, 2, 2, 3, 3, 3, 4
13. 19, 25, 31, 19, 34, 22, 31, 34
14. 58, 58, 60, 60, 60, 61, 63

15. **Sports** Lamont bowled 153, 145, 148, and 158 in four games. Use the mean, median, and mode of Lamont's bowling scores to answer each question.
   a. Which value describes Lamont's average score?
   b. Which value best describes Lamont's scores? Explain.

mean = 151
median = 150.5
mode: none

Use the data to make a box-and-whisker plot.

16. 62, 63, 62, 64, 68, 62, 62
17. 85, 90, 81, 100, 92, 85
18. 1, 2, 3, 4, 5, 6, 7, 8
19. 17, 13, 19, 17, 11, 17, 14, 11, 19, 12

Find the mean, median, mode, and range of each data set.

20. 1, 2, 3, 4, 5, 6, 7, 8, 9, 10
21. 5, 6, 6, 5, 5
22. 2.1, 4.3, 6.5, 1.2, 3.4
23. $0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1$
24. 23, 25, 26, 25, 23
25. −3, −3, −3, −2, −2, −1
26. 1, 4, 9, 16, 25, 36
27. 51, 53, 51, 53, 51
28. 1, 0, 0, 0, 1, 1, 4

29. **Estimation** Estimate the mean of $16\frac{7}{8}, 12\frac{1}{4}, 22\frac{1}{10}, 18\frac{5}{7}, 19\frac{1}{3}$, and $13\frac{8}{11}$.

30. **Weather** The high temperatures in degrees Fahrenheit on 11 consecutive days were 68, 71, 75, 74, 77, 71, 73, 71, 72, 74, and 79. Find the mean, median, mode, and range of the temperatures. Then find the mean, median, mode, and range of the temperatures if the next day's temperature was 70°F. Describe the effect on the mean, median, mode, and range.
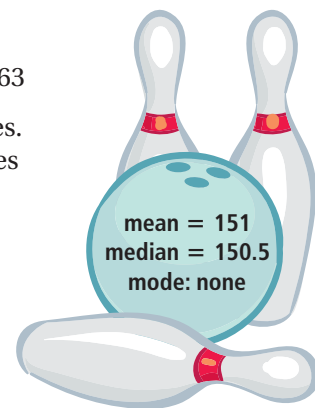
31. This problem will prepare you for the Multi-Step TAKS Prep on page 574.

In the 2004 Olympic games in Athens, the following results occurred for the men's pole vault finals: 5.95, 5.90, 5.85, 5.80, 5.75, 5.75, 5.75, 5.65, 5.65, 5.65, 5.55, 5.55, 5.55, 5.55, 5.55, 5.55. The results are heights in meters.

a. Find the mean, median, mode and range of this data set.

b. The gold medal was won by Timothy Mack of the United States. What was his height in the pole vault event?

c. Which measure of central tendency best describes the data set? Explain.

32. **Business** The salaries for eight people working for a small company are shown in the table. Determine the mean and median salaries. Which measure of central tendency best describes a typical salary of an employee at this company? Explain.

Salaries ($)	
20,000	25,000
20,000	30,000
23,000	35,000
25,000	100,000

Use the data to make a box-and-whisker plot.

33. 25, 28, 26, 16, 18, 15, 25, 28, 26, 16

34. 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31

35. 1, 1, 1, 1, 2, 2, 2, 2, 3, 3, 4, 4, 4, 4, 4

36. **Sports** The table shows the attendance at 7 football games at Jefferson High School. Which measure of central tendency best shows the typical attendance at a football game?

Attendance at Football Games	
Eagles vs. Bulldogs	743
Eagles vs. Panthers	768
Eagles vs. Coyotes	835
Eagles vs. Bears*	1218
Eagles vs. Colts	797
Eagles vs. Mustangs	854

*Homecoming Game

37. **Write About It** Explain how an outlier with a large value will affect the mean. Explain how an outlier with a small value will affect the mean.

38. Allison has taken 5 tests worth 100 points each. Her scores are shown in the gradebook below. What score does she need on her next test to get an average of 90%?

Student	Test 1	Test 2	Test 3	Test 4	Test 5	Test 6	Average
Allison	88	85	89	92	90		

Tell whether each statement is *sometimes*, *always*, or *never* true.

39. The mean is a value in the data set.

40. The median is a value in the data set.

41. The mode is a value in the data set.

42. The mean is affected by including an outlier.

**43.** The mode is affected by including an outlier.

44. Critical Thinking Consider the given data set: 1, 2, 3, 5, 8, 13, 21.

   a. Find the mean of the given data set.

   b. What happens to the mean of the data set if every number is increased by 2?

   c. What happens to the mean of the data set if every number is multiplied by 2?

TEST PREP

45. Which value must be represented on a box-and-whisker plot?

   (A) Mean   (B) Median   (C) Mode   (D) Range

46. Which value must be a value in a data set?

   (F) Mean   (G) Median   (H) Mode   (J) Range

47. Which of the following could be used to find the mean, median, mode, and range of a data set?

   (A) Histogram   (C) Stem-and-leaf plot

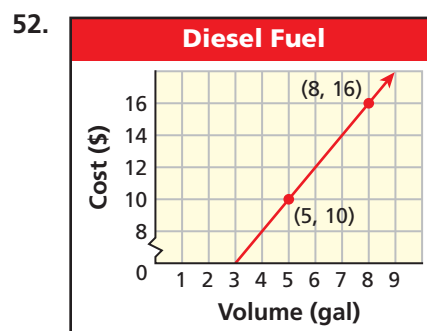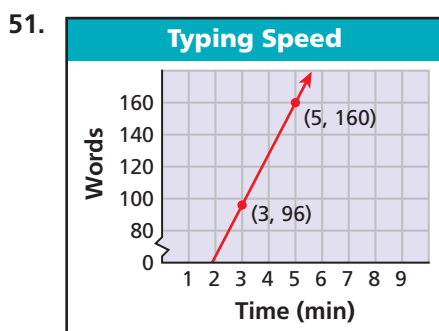   (B) Frequency table   (D) Box-and-whisker plot

CHALLENGE AND EXTEND

48. List a set of data values with the following measures of central tendency:
mean = 8   median = 7   mode = 6

49. For the box-and-whisker plot at right, how does the range of the lower half of the data differ from the range of the upper half of the data?

50. List a set of data values that can be represented by the box-and-whisker plot at right.
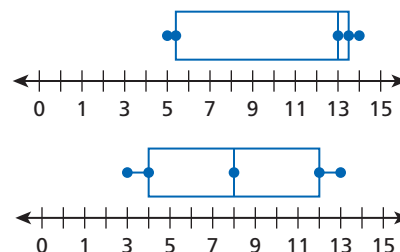
SPIRAL REVIEW

Find the slope of each line. Tell what rate the slope represents. *(Lesson 5-3)*

51.

Typing Speed — (5, 160), (3, 96); Words vs Time (min)

52.

Diesel Fuel — (8, 16), (5, 10); Cost ($) vs Volume (gal)

53. The length of a rectangle is one less than two times the width. The area is 15 yd². What are the dimensions of the rectangle? *(Lesson 9-6)*

54. The ages of the applicants for a driver's license one day are shown in the table. Create a stem-and-leaf plot of the data. *(Lesson 10-2)*

Ages of Applicants														
17	16	21	16	16	17	35	16	18	17	16	16	23	18	30

10-3

Technology LAB

Use Technology to Make Graphs

You can use a spreadsheet program to create bar graphs, line graphs, and circle graphs. You can also use a graphing calculator to make a box-and-whisker plot.

Use with Lesson 10-3

Activity 1

go.hrw.com/A1/TX
Lab Resources Online
KEYWORD: MA7 Lab10

Many colors are used on the flags of the 50 United States. The table shows the number of flags that use each color. Use a spreadsheet program to make a bar graph to display the data.

Color	Black	Blue	Brown	Gold	Green	Purple	Red	White
Number	27	46	20	36	24	4	34	42

**1** Enter the data from the table in the first two columns of the spreadsheet.

2 Select the cells containing the titles and the data.

Then click the Chart Wizard icon, 📊. Click Column from the list on the left, and then choose the small picture of a vertical bar graph. Click Next.
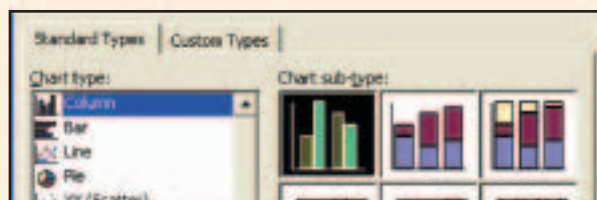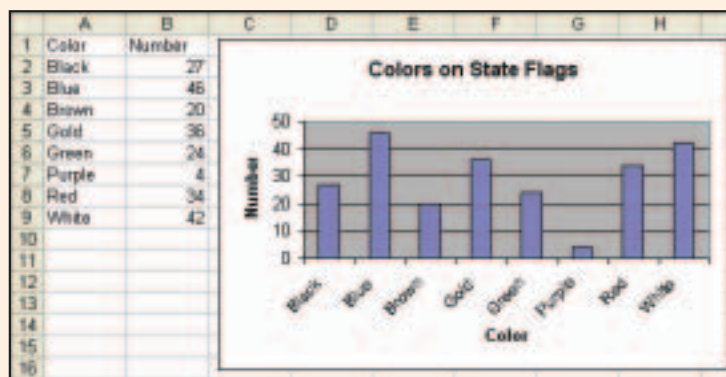
3 The next screen shows the range of cells used to make the graph. Click Next.

4 Give the chart a title and enter titles for the *x*-axis and *y*-axis. Click the Legend tab, and then click the box next to Show Legend to turn off the key. (A key is needed when making a double-bar graph.) Click Next.
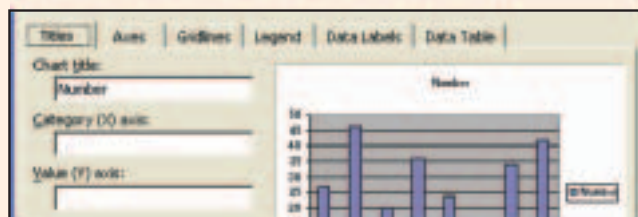
5 Click Finish to place the chart in the spreadsheet.

1. The table shows the average number of hours of sleep people at different ages get each night. Use a spreadsheet program to make a bar graph to display the data.
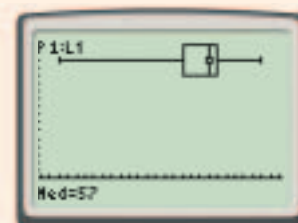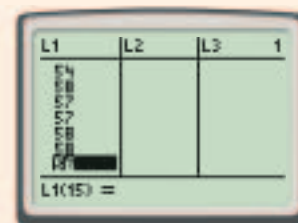
Age (yr)	3–9	10–13	14–18	19–30	31–45	46–50	51+
Sleep (h)	11	10	9	8	7.5	6	5.5

Activity 2

Adrianne is a waitress at a restaurant. The amounts Adrianne made in tips during her last 15 shifts are listed below. Use a graphing calculator to make a box-and-whisker plot to display the data. Give the minimum, first quartile, median, third quartile, and maximum values.

$58, $63, $40, $44, $57, $59, $61, $53, $54, $58, $57, $57, $58, $58, $56

1 To make a list of the data, press **STAT**, select **Edit**, and enter the values in List 1 **(L1)**. Press **ENTER** after each value.

2 To use the **STAT PLOT** editor to set up the box-and-whisker plot, press **2nd** **Y=** , and then **ENTER**.

Press **ENTER** to select **Plot 1**.

3 Select **On**. Then use the arrow keys to choose the fifth type of graph, a box-and-whisker plot.

Xlist should be **L1** and **Freq:** should be 1.

4 Press **ZOOM** and select **9: ZoomStat** to see the graph in the statistics window.

5 Use **TRACE** and the arrow keys to move the cursor along the graph to the five important values: minimum **(MinX)**, first quartile **(Q1)**, median **(MED)**, third quartile **(Q3)**, and maximum **(MaxX)**.

minimum: 40

first quartile: 54

median: 57

third quartile: 58

maximum: 63

2. The average length in inches of the ten longest bones in the human body are listed. Use a graphing calculator to make a box-and-whisker plot to display the data. What are the minimum, first quartile, median, third quartile, and maximum values of the data set?

19.88, 16.94, 15.94, 14.35, 11.10, 10.40, 9.45, 9.06, 7.28, 6.69

10-4 Misleading Graphs and Statistics

Objectives
Recognize misleading graphs.

Recognize misleading statistics.

Vocabulary
random sample

Why learn this?
A misleading graph can be used to distort the results of a student council election. (See Example 3.)

Graphs can be used to influence what people believe. The way data is displayed can influence how the data is interpreted.

EXAMPLE 1 **Misleading Bar Graphs**

The graph shows the size of tomatoes on plants that were treated with different fertilizers.

A Explain why the graph is misleading.
The scale on the vertical axis begins at 80. This exaggerates the differences between the sizes of the bars.

B What might someone believe because of the graph?
Someone might believe that the tomato treated with fertilizer D is much larger than the other tomatoes. It is only 3.3 grams larger than the tomato treated with fertilizer B.

Effect of Fertilizer

82.8 g   81.9 g   83.4 g   85.2 g

A   B   C   D
Fertilizer
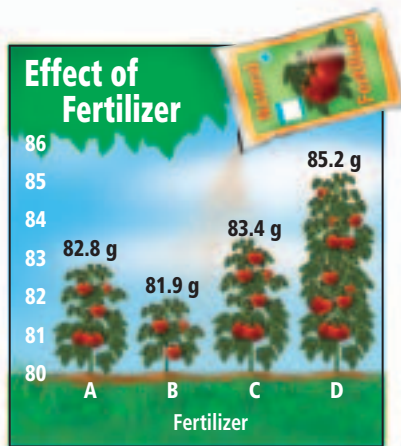
CHECK IT OUT!
1. Who might want to use the graph above? Explain.

EXAMPLE 2 **Misleading Line Graphs**

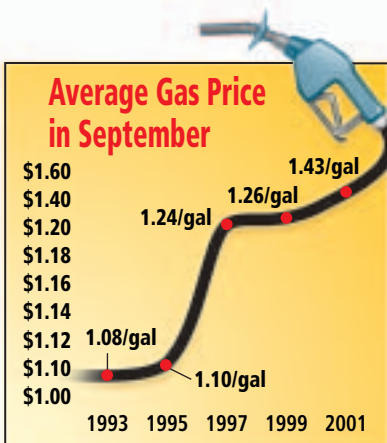The graph shows the average price of gasoline in the U.S. in September.

A Explain why the graph is misleading.
The intervals on the vertical axis are not equal.

B What might people be influenced to believe by the graph?
Someone might believe that the price of gasoline increased the most between 1995 and 1997. However, the change between 1995 and 1997 was only $0.14/gal while the change between 1999 and 2001 was $0.17/gal.

Average Gas Price in September

$1.60
$1.40
$1.20
$1.18
$1.16
$1.14
$1.12
$1.10
$1.00

1.43/gal
1.26/gal
1.24/gal
1.08/gal
1.10/gal

1993   1995   1997   1999   2001

CHECK IT OUT!
2. Who might want to use the graph above? Explain.

A circle graph compares each category of a data set to the whole. When any category is not represented in the graph, it may appear that another category represents a greater percentage of the total than it should.

EXAMPLE 3 **Misleading Circle Graphs**

The graph shows what percent of the total votes were received by three candidates for student council president.

A Explain why the graph is misleading.

The sections of the graph do not add to 100%, so the votes for at least one of the candidates is not represented.

Student Council Election

Atkins 10%
Smith 20%
Napier 5%

B What might people be influenced to believe by the graph?

Someone might believe that Smith won the election.
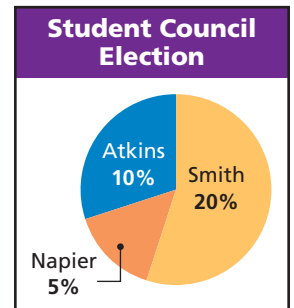
CHECK IT OUT!
3. Who might want to use the graph above? Explain.

Statistics can be misleading because of the way the data is collected or the way the results are reported. A *random sample* is a good way to collect unbiased data. In a **random sample**, all members of the group being surveyed have an equal chance of being selected.

EXAMPLE 4 **Misleading Statistics**

A researcher surveys people leaving a basketball game about what they like to watch on TV. Explain why the following statement is misleading: "80% of people like to watch sports on TV."
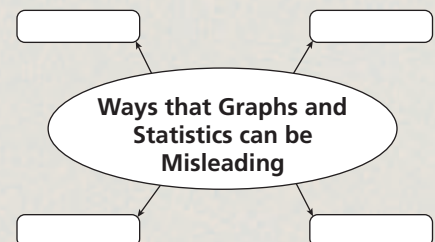
The sample is biased because people who attend sporting events are more likely to watch sports on TV than people who watch TV but do not attend sporting events.

CHECK IT OUT!
4. A researcher asks 4 people if they have seasonal allergies. Three people respond yes. Explain why the following statement is misleading: "75% of people have seasonal allergies."

THINK AND DISCUSS

1. Give an example of a situation in which someone might intentionally try to make a graph misleading.

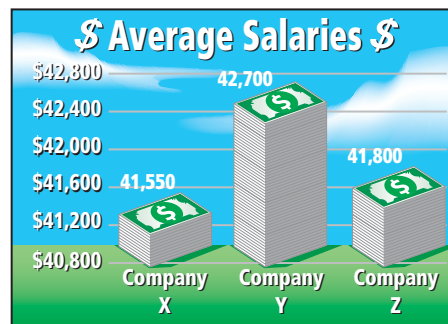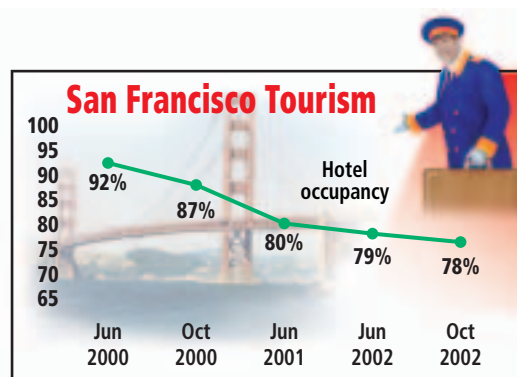**2.** **GET ORGANIZED** Copy and complete the graphic organizer. Add more boxes if needed.

Ways that Graphs and Statistics can be Misleading

TAKS | Grade 8 Obj. 5, 6
Grades 9–11 Obj. 1, 2, 9, 10

go.hrw.com/A1/TX
Homework Help Online
KEYWORD: MA7 10-4
Parent Resources Online
KEYWORD: MA7 Parent

GUIDED PRACTICE

1. **Vocabulary** Explain in your own words what the term *random sample* means.

SEE EXAMPLE **1**
p. 702

2. The graph shows the average salaries of employees at three companies.
   a. Explain why the graph is misleading.
   b. What might someone believe because of the graph?
   c. Who might want to use this graph?

$ Average Salaries $
$42,800
$42,400   42,700
$42,000
$41,600 41,550   41,800
$41,200
$40,800 Company   Company   Company
           X   Y   Z

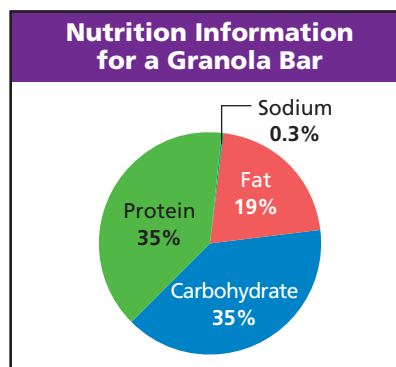SEE EXAMPLE **2**
p. 702

3. The graph shows hotel occupancy in San Francisco over four years.
   a. Explain why the graph is misleading.
   b. What might someone believe because of the graph?
   c. Who might want to use this graph?

San Francisco Tourism
100
95
90   92%   Hotel
85   occupancy
80   87%
75   80%
70   79%   78%
65
     Jun   Oct   Jun   Jun   Oct
     2000   2000   2001   2002   2002

SEE EXAMPLE **3**
p. 703

4. The graph shows the nutritional information for a granola bar.
   a. Explain why the graph is misleading.
   b. What might someone believe because of the graph?
   c. Who might want to use this graph?

Nutrition Information for a Granola Bar
Sodium 0.3%
Fat 19%
Protein 35%
Carbohydrate 35%

SEE EXAMPLE **4**
p. 703

5. Three students were surveyed about their favorite teacher. Two students answer Mr. Gregory, and one answers Mr. Blaine. Explain why the following statement is misleading: "Mr. Gregory is the favorite teacher of a majority of the students."

6. A researcher surveys people at a shopping mall about whether they favor enlarging the size of the mall parking lot. Explain why the following statement is misleading: "85% of the community is in favor of enlarging the parking lot."

## PRACTICE AND PROBLEM SOLVING

TEKS ★ TAKS

Skills Practice p. S22
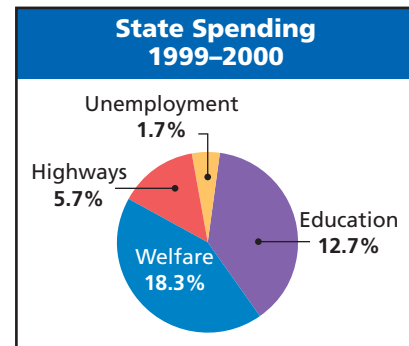
Application Practice p. S37

7. The graph shows the median rent for men and women in a metropolitan area.

 a. Explain why the graph is misleading.

 b. What might someone believe because of the graph?

 c. Who might want to use this graph?

Metropolitan Area Median Rent 2000

$620
$612
$610
$601
$600
$590
$580
$570

Single man   Single woman
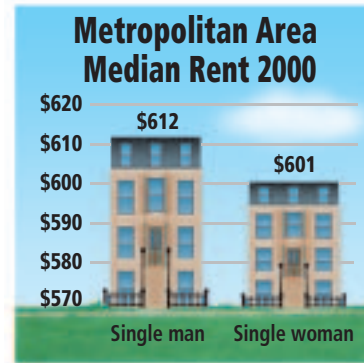
8. The graph shows the export prices of Colombian arabica coffee over nine years.

 a. Explain why the graph is misleading.

 b. What might someone believe because of the graph?

 c. Who might want to use this graph?

Colombian Arabica Coffee
Export Price/lb

$5.56
$4.60
$3.64
$2.68
$1.72
$0.49
$0   1994 1996 1998 2000 2002
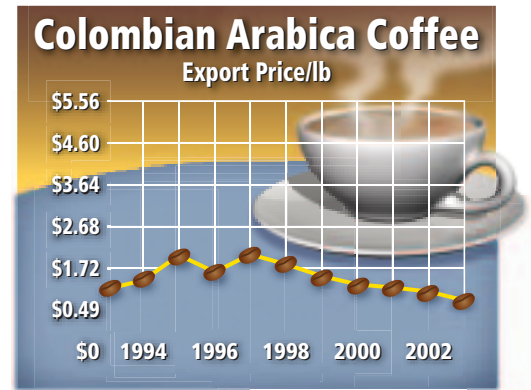
9. The graph shows how the state spent tax dollars during 1999 and 2000.

 a. Explain why the graph is misleading.

 b. What might someone believe because of the graph?

 c. Who might want to use this graph?

10. A college math course has one section with 240 students and 8 sections with 30 students. Explain why the following statement is misleading: "The average class size for the course is 53 students."

State Spending 1999–2000

Unemployment 1.7%
Highways 5.7%
Welfare 18.3%
Education 12.7%

MULTI-STEP TAKS PREP

**11.** This problem will prepare you for the Multi-Step TAKS Prep on page 710.

The table shows scores from the women's gymnastics finals in the floor exercise at the 2004 Summer Olympic Games.

 a. Find the average score for the women in the finals.

 b. Why would it be misleading to say that this value is the average for women in the floor exercise?

 c. Make a graph for this data that could convince someone that the difference between the first place score and the eighth place score was very small.

Rank	Name	Score
1	Catalina Ponor	9.750
2	Nicoleta Sofronie	9.562
3	Patricia Moreno	9.487
4	Fei Cheng	9.412
5	Daiane dos Santos	9.375
6	Mohini Bhardwaj	9.312
7	Kate Richardson	9.312
8	Alina Kozich	8.500

12. ///**ERROR ANALYSIS**///The graph shows the population of a city over time. Which conclusion is incorrect? Explain why the conclusion is incorrect and how the graph was misleading.

A The population has grown rapidly over 20 years.

B The population has grown slowly over 20 years.

City Population
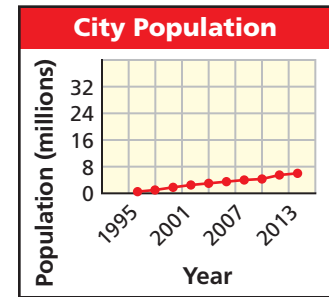
13. The table shows the average connection speeds of some broadband Internet service providers.

Provider	Connection Speed (Kbps)
Speedy Online	954
TelQuick	914
Alacrity	858

   a. Construct a display that suggests that Speedy Online is much faster than the other services.

   b. Construct a display that suggests that all of the services offer about the same connection speeds.

   c. **Write About It** Where might you expect to see your graph from part **b**? Explain.

14. **Critical Thinking** Explain how a graph can show truthful data but still be misleading.

TEST PREP

15. What might someone be influenced to believe because of the graph?

   Ⓐ The measles vaccine was introduced when the mortality rate was at its highest.

   Ⓑ The measles vaccine was unnecessary.

   Ⓒ The measles vaccine dramatically decreased the mortality rate.

   Ⓓ The measles vaccine increased the mortality rate.

United States Measles Mortality Rates
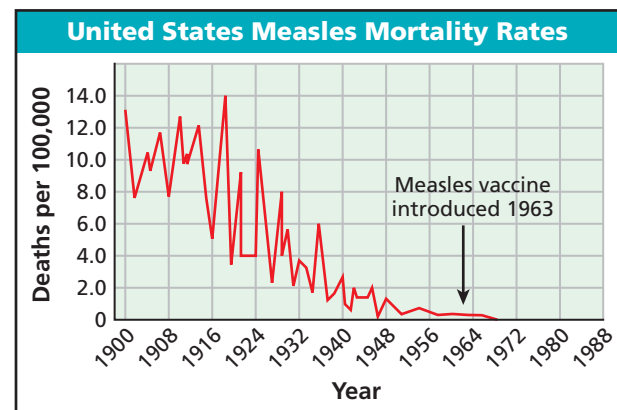
Source: www.healthsentinel.com

16. The table shows the number of votes cast in the 2000 U.S. presidential election and in the 2002 French presidential election. What additional information is needed to determine whether the following statement is misleading?

Country	Votes Cast
United States	105,405,100
France	29,497,272

"American voters are more likely to vote than French voters."

   Ⓕ The number of candidates in each election

   Ⓖ The legal voting age in France

   Ⓗ The number of registered voters in the United States in 2000 and France in 2002

   Ⓙ The number of polling locations in the United States in 2000 and France in 2002

CHALLENGE AND EXTEND

17. Logic A fingerprint analyst is studying a fingerprint that was found in the chemistry lab. He reports that the fingerprint belongs to Dr. Arenson. Below are two questions the analyst was asked and the answers he gave.

Question 1: What are the chances that the fingerprint belongs to someone else who has the same fingerprint as Dr. Arenson?

Answer: One in several billion.

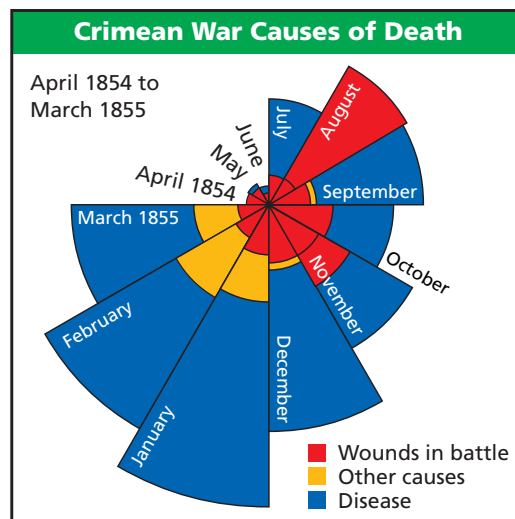**Question 2:** What are the chances that the fingerprint was wrongly identified?

Answer: About 1 in 100.

a. What is the difference between the two questions?

b. What does the answer to question 1 lead you to believe?

c. Who do you think might have asked question 1?

d. What does the answer to question 2 lead you to believe?

e. Who do you think might have asked question 2?

18. History Graphs like the one at right were created by Florence Nightingale. Nightingale served as a nurse during the Crimean War and was concerned with the unsanitary conditions the soldiers lived in. Each "wedge" of the circle represents a month between April 1854 and March 1855.

a. What do you think Florence Nightingale wanted to show with this graph?

b. Who do you think Nightingale showed the graph to?

Crimean War Causes of Death

April 1854 to March 1855

■ Wounds in battle
■ Other causes
■ Disease

Source: The Florence Nightingale Museum

SPIRAL REVIEW

Write an inequality for each situation. *(Lesson 3-1)*

19. The maximum weight for a certain truck load is 1500 pounds.

20. Isaac's research paper must be at least 12 pages.

21. A moving company will transport up to 20 boxes for no fee.

Solve each inequality and graph the solutions. *(Lesson 3-4)*

22. $2x - 3 < 7$ **23.** $3(t - 1) \geq -15$ **24.** $6 - n < 2n + 9$

25. The table shows the weight of a golden retriever at different ages. Choose a type of graph to display the given data. Make the graph, and explain why you chose that type of graph. *(Lesson 10-1)*

Age (mo)	1	3	6	12
Weight (lb)	8	23	45	66

Sampling and Bias

Data Analysis

If you wanted to collect data about a very large group of people, you would need to survey a smaller group. The large group that contains all the people you could survey is called a *population*. The smaller group is called a *sample*.

You have learned that a random sample is a good way of collecting data that is unbiased. There are different ways of selecting a random sample.

Random Samples		
Type	**Definition**	**Example**
Simple Random Sample	Every member of the population has an equal chance of being chosen for the sample.	The names of all students in your class are placed in a hat and three names are chosen at random.
Stratified Random Sample	The population is divided into similar groups. Then a simple random sample is chosen from each group.	Your class is divided into boys and girls and two students are chosen at random from each group.
Systematic Random Sample	A member of the population is chosen for the sample at a regular interval.	Every third student who comes into the classroom is chosen.

Example 1

In each situation, identify the population and the sample. Tell whether each sample is a simple, stratified, or systematic random sample.

A **For one week, the manager of a pet supplies store asks every tenth customer what brand of pet food they buy.**

population: all customers at the pet supplies store during one week

sample: every tenth customer during that same week

The sample is systematic because one member of the population is chosen for the sample at a regular interval.

B **Every person who enters a theater one evening places their ticket stub in a bowl. The theater owner chooses five ticket stubs to award prizes.**

population: all people who put their ticket stub in the bowl

sample: five ticket stubs chosen from the bowl

The sample is simple because every member of the population has the same chance of being chosen.

C **One student from each classroom at a school is chosen at random for a committee.**

population: all students at a school

sample: one student from each classroom

The sample is stratified because the population is divided into similar groups and one member is chosen at random from each group.

1. Choose a topic to research. Describe the population, why you need to choose a sample, and the sample.
2. Choose whether to use a simple, stratified, or systematic random sample. Explain your choice and the process you would use to choose your sample.

A random sample is not biased because no part of the population is favored over another. In a biased sample, one or more parts of the population have an advantage for being chosen for the sample.

There are two main types of biased samples.

Biased Samples		
Type	**Definition**	**Example**
Convenience Sample	Those members of the population that are easily accessed are chosen for the sample.	A reporter questions people he personally knows.
Voluntary Response Sample	Members of the population who want to participate make up the sample.	A reporter questions people who fill out comment cards and indicate that they would like to be contacted.

Example 2

In each situation, identify the population and the sample. Tell whether each sample is a convenience or voluntary response sample. Explain why the sample is biased.

A **A website asks visitors to complete a survey about internet usage.**

population: all visitors to the website

sample: those visitors who choose to complete the survey

The sample is a voluntary response sample because visitors to the website chose whether to complete the survey. The sample is biased because those visitors who choose to complete the survey may not be representative of all visitors to the website.

B **A reporter asks people leaving a shopping center through one door about their shopping habits.**

population: all people at the shopping center

sample: people who leave from one door at the shopping center

The sample is a convenience sample because the people leaving through the chosen door are easily accessed. The sample is biased because people leaving through another door do not have an opportunity to be chosen.

3. Choose a topic to research. Describe the population, why you need to choose a sample, and the sample.
4. Assume that you want an unbiased sample. Choose whether to use a simple, stratified, or systematic random sample. Explain your choice and the process you would use to choose your sample.
5. Describe how a biased sample could be chosen for this same situation.

MULTI-STEP TAKS PREP

TAKS | Grade 8 Obj. 5
Grades 9–11 Obj. 9
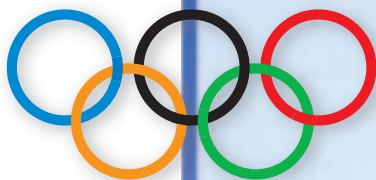
Data Analysis

USA! USA! USA! In 708 B.C.E., the Olympic Games included the pentathlon. This event included the discus, javelin, long jump, running, and wrestling. The winner of the pentathlon was considered a "super athlete." The modern pentathlon of the Olympic Games includes five disciplines: target shooting, fencing, swimming, horseback riding, and cross-country running.

1. The pentathlon includes five events. What type of graph should be used to show what percent of this event is made up of each discipline?

2. Do you think the winner of the modern pentathlon must win each of the five events? Explain.

3. Thirty-two men competed in the modern pentathlon in the 2004 Olympic Games. The chart shows the total points for the competitors.

Pentathlon Scores
5480 5428 5392 5356 5340
5332 5320 5295 5276 5252 5200 5196
5192 5184 5180 5172 5144 5132 5084
5084 5068 5016 4976 4936 4932 4916
4904 4848 4732 4676 4420 4388

Find the mean, median, and mode of this data set. Which value best describes the data? Explain.

4. Find the minimum, first quartile, third quartile, and maximum values of the data set above. Use these values and the median to make a box-and-whisker plot of the data.

Quiz for Lessons 10-1 Through 10-4

10-1 Organizing and Displaying Data

Don's Recyclables (lb)

Plastic 1 — Metals 3 — Paper/cardboard 11 — Glass 5

Use the circle graph for Problems 1–3.

1. Which material represents over 50% of Don's recyclables?

2. How many pounds of materials does Don recycle?

3. What percent of Don's recyclables are glass?

4. The table shows the total proceeds for a fund-raiser at various times during the day. Choose a type of graph to display the given data. Make the graph, and explain why you chose that type of graph.
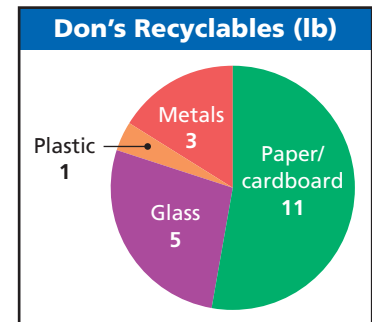
Time	Total Proceeds (thousand $)	Time	Total Proceeds (thousand $)
3:00 P.M.	1.5	6:00 P.M.	6.5
4:00 P.M.	2	7:00 P.M.	8
5:00 P.M.	4	8:00 P.M.	9.5

10-2 Frequency and Histograms

5. The Miller family is going to buy a new car. The approximate prices of the cars they are considering are given. Use the data to make a stem-and-leaf plot.

$24,000, $28,000, $26,000, $32,000, $30,000, $41,000, $27,000, $22,000, $26,000, $33,000

6. The number of people at a caterer's last 12 parties are given below.

16, 18, 17, 19, 15, 25, 18, 17, 18, 16, 17, 19

a. Use the data to make a frequency table with intervals.

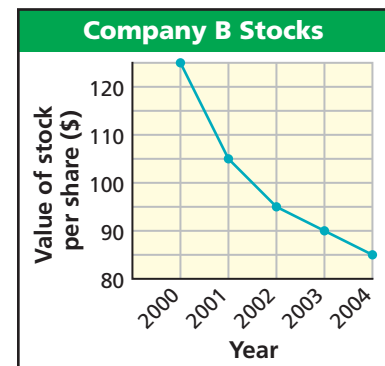b. Use your frequency table from part a to make a histogram.

10-3 Data Distributions

7. The daily high temperatures on 14 consecutive days in one city were 59°F, 49°F, 48°F, 46°F, 47°F, 51°F, 49°F, 43°F, 45°F, 52°F, 51°F, 51°F, 51°F, and 38°F.

a. Find the mean, median, and mode of the temperatures.

b. Which value describes the average high temperature for the 14 days?

c. Which value best describes the high temperatures? Explain.

8. Use the temperature data above to make a box-and-whisker plot.

10-4 Misleading Graphs and Statistics

9. The graph shows the value of a company's stock over time. Explain why the graph is misleading. What might people believe because of the graph? Who might want to use this graph?

10. The results of an online survey of 230 people showed that 92% of the population felt very comfortable using technology. Explain why this statistic is misleading.

Company B Stocks

Value of stock per share ($): 80, 90, 100, 110, 120

Year: 2000, 2001, 2002, 2003, 2004

Algebra LAB

Simulations

A simulation can be used to model an experiment that would be difficult or inconvenient to actually perform. In this lab, you will conduct simulations.

Use with Lesson 10-5

⭐ **TEKS A.2.D Foundations for functions* collect and organize data….**

Activity

The local movie theater is offering an opportunity for customers to win a free night at the movies. To win, you must collect six different letters to spell CINEMA. Each movie ticket sold during this promotion will have one of the six letters stamped on the back of the ticket. An equal number of tickets will be stamped with each of the letters.

1 Since there are six different letters that appear on the tickets an equal number of times, you can use a number cube to simulate collecting the six letters.

Each of the numbers on the number cube will represent a letter. Each roll of the number cube will represent purchasing one movie ticket, and the number rolled will represent the letter stamped on the ticket.

C I N E M A
1 2 3 4 5 6
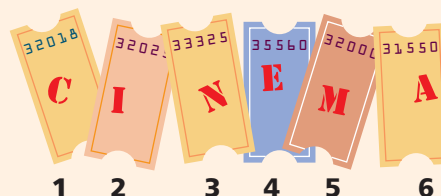
2 The table shows the results of rolling the number cube until each number has been rolled once.
  a. Based on the results shown in the table, how many rolls did it take to get all six numbers?
  b. Based on the results in the table, how many movie tickets would you have to buy to get all six letters? If you purchased this number of tickets, would you be sure to win? Explain.

Number on Cube	Letter on Ticket	Frequency
1	C	I
2	I	JHT I
3	N	IIII
4	E	II
5	M	II
6	A	III

Try This

1. Repeat the simulation four more times and record the results.

2. Find the average number of rolls from all five simulations $\left(\dfrac{\text{total number of rolls from 5 simulations}}{5}\right)$.

3. Based on your answer to Problem 2, how many movie tickets would you have to buy to get all six letters? Is this number different from the answer you gave based on the results in the table above?

4. Would any of your answers have been different if you had used a different correspondence between the numbers and letters? Explain.

Experimental Probability

Objectives
Determine the experimental probability of an event.

Use experimental probability to make predictions.

Vocabulary
experiment
trial
outcome
sample space
event
probability
experimental probability
prediction

Why learn this?
Experimental probability can be used by manufacturers for quality control. (See Example 4.)

An **experiment** is an activity involving chance. Each repetition or observation of an experiment is a **trial**, and each possible result is an **outcome**. The **sample space** of an experiment is the set of all possible outcomes.

Experiment	Rolling a number cube	Tossing a coin	Spinning a game spinner
Sample Space	{1, 2, 3, 4, 5, 6}	{heads, tails}	{red, blue, green, yellow}

EXAMPLE 1 **Identifying Sample Spaces and Outcomes**

Identify the sample space and the outcome shown for each experiment.

A tossing two coins
Sample space: {HH, HT, TH, TT}
Outcome shown: heads, tails (H, T)

B spinning a game spinner
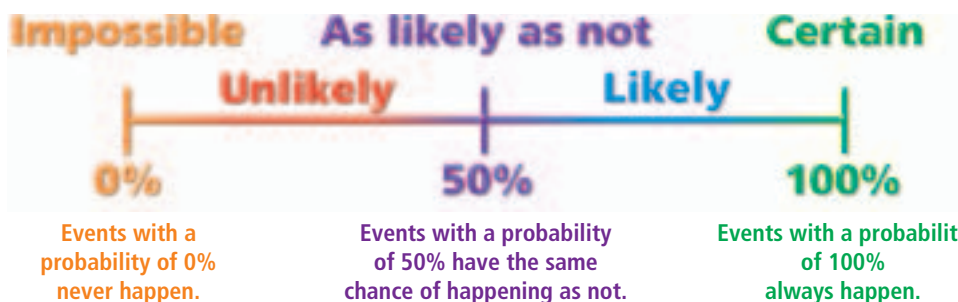Sample space: {yellow, red, blue, green}
Outcome shown: green

CHECK IT OUT!

1. Identify the sample space and the outcome shown for the experiment: rolling a number cube.

An **event** is an outcome or set of outcomes in an experiment. **Probability** is the measure of how likely an event is to occur. Probabilities are written as fractions or decimals from 0 to 1, or as percents from 0% to 100%.

Impossible   **As likely as not**   **Certain**
Unlikely   **Likely**
0%   50%   100%

Events with a probability of 0% never happen.

Events with a probability of 50% have the same chance of happening as not.

Events with a probability of 100% always happen.

EXAMPLE 2 **Estimating the Likelihood of an Event**

Write *impossible, unlikely, as likely as not, likely,* or *certain* to describe each event.

A There are 31 days in August.

August always has 31 days. This event is *certain*.

B Carlos correctly guesses a number between 1 and 1000.

Carlos must pick one outcome out of 1000 possible outcomes. This event is *unlikely*.

C A coin lands heads up.

Heads is one of two possible outcomes. This event is *as likely as not*.

D Cecilia rolls a 10 on a standard number cube.

A standard number cube is numbered 1 through 6. This event is *impossible*.

2. Write *impossible, unlikely, as likely as not, likely,* or *certain* to describe the event: Anthony rolls a number less than 7 on a standard number cube.

You can estimate the probability of an event by performing an experiment. The **experimental probability** of an event is the ratio of the number of times the event occurs to the number of trials. The more trials performed, the more accurate the estimate will be.

Experimental Probability

$$\text{experimental probability} = \frac{\text{number of times the event occurs}}{\text{number of trials}}$$

EXAMPLE 3 **Finding Experimental Probability**

An experiment consists of spinning a spinner. Use the results in the table to find the experimental probability of each event.

Outcome	Frequency
Red	7
Blue	8
Green	5

A spinner lands on blue

$$\frac{\text{number of times the event occurs}}{\text{number of trials}} = \frac{8}{7 + 8 + 5}$$
$$= \frac{8}{20}$$
$$= \frac{2}{5}$$

B spinner does not land on green

When the spinner does not land on green, it must land on red or blue.

$$\frac{\text{number of times the event occurs}}{\text{number of trials}} = \frac{7 + 8}{7 + 8 + 5}$$
$$= \frac{15}{20}$$
$$= \frac{3}{4}$$

Helpful Hint

Probabilities can be expressed as fractions, decimals, or percents. For Example 3, the probabilities are:

3A: $\frac{2}{5} = 0.4 = 40\%$

3B: $\frac{3}{4} = 0.75 = 75\%$

CHECK IT OUT! Use the information in Example 3 to find the experimental probability of each event.

3a. spinner lands on red

3b. spinner does not land on red

You can use experimental probability to make *predictions*. A **prediction** is an estimate or guess about something that has not yet happened.

EXAMPLE 4 *Quality Control Application*

A manufacturer inspects 800 light bulbs and finds that 796 of them have no defects.

A What is the experimental probability that a light bulb chosen at random has no defects?

Find the experimental probability that a light bulb has no defects.

$$\frac{\text{number of times the event occurs}}{\text{number of trials}} = \frac{796}{800}$$

$$= 99.5\%$$

The experimental probability that a light bulb has no defects is 99.5%.

B The manufacturer sent a shipment of 2400 light bulbs to a retail store. Predict the number of light bulbs in the shipment that are likely to have no defects.

Find 99.5% of 2400.

$$0.995(2400) = 2388$$

The manufacturer predicts that 2388 light bulbs have no defects.

CHECK IT OUT! **4.** A manufacturer inspects 1500 electric toothbrush motors and finds 1497 to have no defects.

a. What is the experimental probability that a motor chosen at random will have no defects?

b. There are 35,000 motors in a warehouse. Predict the number of motors that are likely to have no defects.

THINK AND DISCUSS

1. Explain the difference between an outcome and an event.

2. Is the experimental probability of an event always the same? Explain why or why not.

3. GET ORGANIZED Copy and complete the graphic organizer. In each box, write an example of an event that has the given likelihood.

Know it! Note

Likelihood of an Event

| Impossible | Unlikely | As likely as not | Likely | Certain |
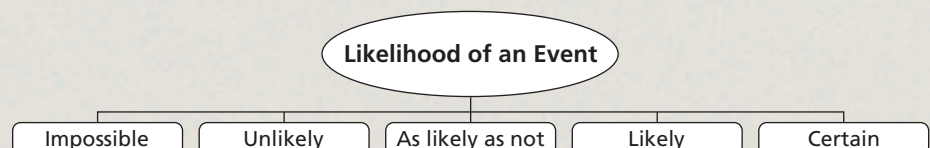
GUIDED PRACTICE

1. **Vocabulary** Give an example of an *event* that has two possible *outcomes*.

SEE EXAMPLE **1**
p. 713

Identify the sample space and the outcome shown for each experiment.

2. rolling a number cube
3. spinning a spinner
4. tossing 3 coins

SEE EXAMPLE **2**
p. 714

Write *impossible, unlikely, as likely as not, likely*, or *certain* to describe each event.

5. Peter was born in January. Thomas was born in June. Peter and Thomas have the same birthday.

6. The football team won 9 of its last 10 games. The team will win the next game.

7. A board game has a rule that if you roll the game cube and get a 6, you get an extra turn. You get an extra turn on your first roll.

SEE EXAMPLE **3**
p. 714

An experiment consists of rolling a number cube. Use the results in the table to find the experimental probability of each event.

Outcome	1	2	3	4	5	6
Fequency	5	6	2	2	3	7

8. rolling a 6

9. rolling an even number

10. not rolling a 6

SEE EXAMPLE **4**
p. 715

11. **Sports** One game of bowling consists of ten frames. Elyse usually rolls 3 strikes in each game.
   a. What is the experimental probability that Elyse will roll a strike on any frame?
   b. Predict the number of strikes Elyse will throw in 18 games.

PRACTICE AND PROBLEM SOLVING

Identify the sample space and the outcome shown for each experiment.

12. tossing two coins
13. spinning a spinner
14. selecting a marble

Independent Practice	
For Exercises	See Example
12–14	1
15–17	2
18–20	3
21	4

TEKS TAKS

Skills Practice p. S23
Application Practice p. S37

Write *impossible, unlikely, as likely as not, likely*, or *certain* to describe each event.

15. Marlo purchased a new pair of shoes. She takes one shoe out of the box. The shoe is for the left foot.

16. Sam takes the bus to school. The bus came late twice in the last two weeks. The bus will be late today.

17. Tammy dropped two quarters on the floor. At least one of them lands heads up.

An experiment consists of randomly choosing a marble from a bag. Use the results in the table to find the experimental probability of each event.

Outcome	Frequency
Red	4
Blue	6
Green	6
Yellow	9

18. choosing a yellow marble

19. choosing a blue marble

20. not choosing a green marble

21. **Sports** A ski lodge inspects 80 skis and finds 4 to be defective.
  a. What is the experimental probability that a ski chosen at random will be defective?
  b. The lodge has 420 skis. Predict the number of skis that are likely to be defective.

22. The table shows the results of a survey asking students the season of their birthday. What is the experimental probability that a student has a birthday during the summer?

Season	Birthdays
Fall	39
Winter	27
Spring	33
Summer	51

23. You and your friend can either go swimming or to a movie on Thursday. The weather forecast says there is a 70% chance of rain on Thursday. Should you plan on going swimming or to a movie? Explain.

24. **Critical Thinking** Tell why it is important to repeat an experiment many times.

25. **Write About It** Explain what it means for an event to have a 50-50 chance of happening.

26. How many outcomes are in the sample space for an experiment consisting of rolling two standard number cubes?

27. **Estimation** A manufacturing company produced 986 units in one day. Of those, 9 units were found to be defective. Estimate the experimental probability that a unit produced that day was defective. Then predict approximately how many units will be defective when 5680 units are produced in one week.

MULTI-STEP TAKS PREP

28. This problem will prepare you for the Multi-Step TAKS Prep on page 744.

In a standard deck of cards, there are 13 cards in each of four suits: hearts, diamonds, clubs, and spades. The hearts and diamonds are red and the clubs and spades are black. Ricardo randomly drew cards from a standard deck of 52 cards. The table shows the results.

Outcome	Frequency
Hearts	7
Diamonds	7
Clubs	8
Spades	6

  a. Find the experimental probability of drawing a club.
  b. Find the experimental probability of drawing a black suit.

29. Alex rolls two standard number cubes. What is the likelihood that the sum of the numbers is less than 4?

Ⓐ Impossible   Ⓑ Unlikely   Ⓒ As likely as not   Ⓓ Likely

30. A community reported that $\frac{2}{3}$ of the residents had a pet and $\frac{1}{2}$ of the pet owners had a dog. If there are 84 residents in the community, how many residents are likely to have a dog?

Ⓕ 14   Ⓖ 28   Ⓗ 42   Ⓙ 56

31. What is the probability that a number chosen at random from the list below will be a solution of the inequality $3x + 2 \leq 23$?

$-8, -7, -6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9$

Ⓐ $\frac{17}{18}$ Ⓑ $\frac{8}{9}$ Ⓒ $\frac{1}{9}$ Ⓓ $\frac{1}{18}$

32. **Short Response** A coin was tossed 50 times. It landed showing heads 6 more times than it landed showing tails. What is the experimental probability of the coin landing on heads? Show your work.

CHALLENGE AND EXTEND

A coin is tossed 3 times. Use the sample space for this experiment to describe each event below as *impossible, unlikely, as likely as not, likely,* or *certain*. Justify each answer.

33. At least 2 heads   **34.** 2 heads and 1 tail

35. 2 tails and 1 head   **36.** 3 tails

37. One coin is tossed 20 times.

   a. The experimental probability of the coin showing heads is 65%. How many times did the coin show tails?

   b. If the coin is tossed ten more times, how many more times must the coin land showing tails for the experimental probability of tails to be 50%?

SPIRAL REVIEW

38. A sales representative earns 4.5% commission on sales. Find the commission earned when the total sales are $124,000. *(Lesson 2-9)*

39. Estimate the tax on a $255 printer when the tax rate is 5.5%. *(Lesson 2-9)*

Compare the graph of each function to the graph of $f(x) = x^2$. *(Lesson 9-4)*

40. $g(x) = \frac{1}{3}x^2$ **41.** $g(x) = -x^2$ **42.** $g(x) = x^2 - 12$

The data shows the number of books read by seven students over the summer:
5, 5, 14, 2, 5, 5, 6. *(Lesson 10-3)*

43. Give the mean, median, and mode of the data.

44. Which measure of central tendency best describes the data? Explain.

45. Create a box-and-whisker plot of the data.

Technology LAB

Use Random Numbers

A calculator can be used to model an experiment that would be difficult or inconvenient to perform. To do this, you will use random numbers.

Use with Lesson 10-5

Activity

You can use a calculator to explore the experimental probability that at least 2 people in a group of 6 people were born in the same month. Assume that all months are equally likely to be a person's birth month.

1 Represent each month with an integer. Since there are 12 months, use the numbers 1–12.

To set your calculator up to generate random numbers, press MATH. Then use the arrow keys to highlight **PRB**. Select **5: randInt(**.
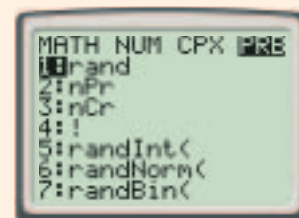
2 Now give the start number, 1, press , , and give the end number, 12.

Each time you press ENTER the calculator will return an integer from 1 to 12.

3 You are considering a group of 6 people. This means you need 6 random numbers.
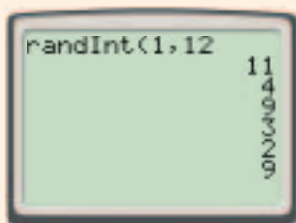
	Person					
	1	**2**	**3**	**4**	**5**	**6**
Trial 1	11	4	9	3	2	9
Trial 2	3	8	10	12	6	2

In the first trial, the number 9 appears twice. This means that two people have a birth day in the ninth month, September.

In the second trial, no number appears more than once. This means that none of the people were born in the same month.

Try This

1. Repeat the experiment until you have 10 trials of the experiment. Count the number of trials in which a number appears more than once. Divide this number by the number of trials, 10, to find the experimental probability that at least 2 people in a group of 6 people will have the same birth month.

2. Gather the results from at least 100 trials of the experiment. (Either perform all of the trials yourself or combine data with your classmates.) Using your results, what is the experimental probability that at least 2 people in a group of 6 people will have the same birth month? Compare the results from 100 trials to the results of 10 trials.

3. How could you set up the experiment to find the experimental probability that at least 2 people in a group of 6 people will have the same birthday (same month and same date)?

Theoretical Probability

Objectives
Determine the theoretical probability of an event.

Convert between probabilities and odds.

Vocabulary
equally likely
theoretical probability
fair
complement
odds

Why learn this?
Theoretical probability can be used to determine the likelihood of different weather conditions. (See Example 2.)

A developing tornado near Amarillo, Texas.

When the outcomes in the sample space of an experiment have the same chance of occurring, the outcomes are said to be **equally likely**.

Equally likely outcomes	Not equally likely outcomes
There is the same chance that the spinner will land on any of the colors.	There is a greater chance that the spinner will land on blue than on any other color.

The **theoretical probability** of an event is the ratio of the number of ways the event can occur to the total number of equally likely outcomes.

> **Theoretical Probability**
>
> $$\text{theoretical probability} = \frac{\text{number of ways the event can occur}}{\text{total number of equally likely outcomes}}$$

An experiment in which all outcomes are equally likely is said to be **fair**. You can usually assume that experiments involving coins and number cubes are fair.

EXAMPLE 1 Finding Theoretical Probability

An experiment consists of rolling a number cube. Find the theoretical probability of each outcome.

A rolling a 3

$$\frac{\text{number of ways the event can occur}}{\text{total number of equally likely outcomes}} = \frac{1}{6}$$

There is one 3 on a number cube.

$$= 0.1\overline{6}$$

$$= 16\frac{2}{3}\%$$

Caution!
The use of the word *experiment* does not necessarily mean you are looking for experimental probability.

An experiment consists of rolling a number cube. Find the theoretical probability of each outcome.

B rolling a number greater than 3

$$\frac{\text{number of ways the event can occur}}{\text{total number of equally likely outcomes}} = \frac{3}{6} = \frac{1}{2}$$ *There are 3 numbers greater than 3.*

$$= 0.5 = 50\%$$

CHECK IT OUT! **An experiment consists of rolling a number cube. Find the theoretical probability of each outcome.**

1a. rolling an even number   **1b.** rolling a multiple of 3

When you toss a coin, there are two possible outcomes, heads or tails. The table below shows the theoretical probabilities and experimental results of tossing a coin 10 times.

Reading Math

The probability of an event can be written as *P*(event). *P*(heads) means "the probability that heads will be the outcome."

	P(heads)	*P*(tails)	*P*(heads) + *P*(tails)
Experimental Probability	$\frac{3}{10}$	$\frac{7}{10}$	$\frac{3}{10} + \frac{7}{10} = \frac{10}{10} = 1$
Theoretical Probability	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2} + \frac{1}{2} = \frac{2}{2} = 1$

The sum of the probability of heads and the probability of tails is 1, or 100%. This is because it is certain that one of the two outcomes will always occur.

$$P(\text{event happening}) + P(\text{event not happening}) = 1$$

The **complement** of an event is all the outcomes in the sample space that are not included in the event. The sum of the probabilities of an event and its complement is 1, or 100%, because the event will either happen or not happen.

$$P(\text{event}) + P(\text{complement of event}) = 1$$

EXAMPLE **2** **Finding Probability by Using the Complement**

The weather forecaster predicts a 20% chance of snow. What is the probability that it will not snow?

$$P(\text{snow}) + P(\text{not snow}) = 100\%$$ *Either it will snow or it will not snow.*

$$20\% + P(\text{not snow}) = 100\%$$

$$\underline{-20\% \qquad\qquad -20\%}$$ *Subtract 20% from both sides.*

$$P(\text{not snow}) = 80\%$$

CHECK IT OUT! **2.** A jar has green, blue, purple, and white marbles. The probability of choosing a green marble is 0.2, the probability of choosing blue is 0.3, the probability of choosing purple is 0.1. What is the probability of choosing white?

Odds are another way to express the likelihood of an event. The *odds in favor of an event* describe the likelihood that the event will occur. The *odds against an event* describe the likelihood that the event will not occur.

Odds are usually written with a colon in the form $a:b$, but can also be written as a to b or $\frac{a}{b}$.

Know it! Note

Odds

ODDS IN FAVOR OF AN EVENT

$$\text{odds in favor} = \frac{\text{number of ways an event can happen}}{\text{number of ways an event can fail to happen}}$$
$$= a:b$$

ODDS AGAINST AN EVENT

$$\text{odds against} = \frac{\text{number of ways an event can fail to happen}}{\text{number of ways an event can happen}}$$
$$= b:a$$

a represents the number of ways an event can occur.
b represents the number of ways an event can fail to occur.

The two numbers given as the odds will add up to the total number of possible outcomes. You can use this relationship to convert between odds and probabilities.

EXAMPLE 3 **Converting Between Odds and Probabilities**

A **The probability of choosing a red card from a standard deck of playing cards is 50%. What are the odds of choosing a red card?**

The probability of choosing a red card is 50%, so the probability of not drawing a red card is 100% − 50% = 50%.

Odds in favor = 50:50, or 1:1

The odds in favor of choosing a red card are 1:1.

B **The odds against choosing a green marble from a bag are 5:3. What is the probability of choosing a green marble?**

The odds against green are 5:3, so the odds in favor of green are 3:5. This means there are 3 favorable outcomes and 5 unfavorable outcomes for a total of 8 possible outcomes.

$$\frac{\text{number of ways event can happen}}{\text{total possible outcomes}} = \frac{3}{8}$$

The probability of choosing a green marble is $\frac{3}{8}$.

Reading Math

You may see an outcome called "favorable." This does not mean that the outcome is good or bad. A favorable outcome is the outcome you are looking for in a probability experiment.
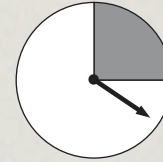
CHECK IT OUT!
3. The odds in favor of winning a free drink are 1:24. What is the probability of winning a free drink?

THINK AND DISCUSS

1. Tell how to find the probability of the complement of an event.

2. **GET ORGANIZED** Copy and complete the graphic organizer using the spinner.

Probabilities on Spinner	
P(gray)	
P(not gray)	
Odds in favor of gray	
Odds against gray	

10-6

Exercises

TAKS Grade 8 Obj. 5, 6
Grades 9–11 Obj. 2, 3, 9, 10

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GUIDED PRACTICE

1. **Vocabulary** All of the outcomes in the sample space that are not included in the event are called the ___?___. (*theoretical probability*, *complement*, or *odds*)

SEE EXAMPLE **1**
p. 720

**Find the theoretical probability of each outcome.**

2. rolling a number divisible by 3 on a number cube

3. flipping 2 coins and both landing with tails showing

4. randomly choosing the letter S from the letters in STARS

5. rolling a prime number on a number cube

SEE EXAMPLE **2**
p. 721

6. A spinner is green, red, and blue. The probability that a spinner will land on green is 15% and red is 35%. What is the probability the spinner will land on blue?

7. The probability of choosing a red marble from a bag is $\frac{1}{3}$. What is the probability of not choosing a red marble?

8. You have a $\frac{1}{50}$ chance of winning. What is the probability you will not win?

9. There is a $\frac{1}{10}$ chance that you will be chosen as class representative. What is the probability that you will not be chosen?
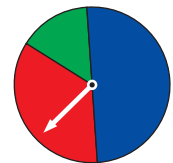
SEE EXAMPLE **3**
p. 722

10. The odds against a spinner landing on blue are 3 : 1. What is the probability of the spinner landing on blue?

11. The probability of choosing an ace from a deck of cards is $\frac{1}{13}$. What are the odds of choosing an ace?

12. The probability of not winning a game is 80%. What are the odds of winning?

13. The odds in favor of a spinner landing on blue are 1 : 3. What is the probability of landing on blue?

PRACTICE AND PROBLEM SOLVING

Find the theoretical probability of each outcome.

14. rolling a 5 on a number cube

15. flipping 2 coins and 1 landing with heads showing, the other with tails showing

16. randomly choosing a blue marble from a bag of 5 blue marbles, 8 red marbles, and 7 yellow marbles

17. The probability of a spinner landing on yellow is $\frac{4}{9}$. What is the probability of it not landing on yellow?

18. There is a 3% probability of winning a game. Find the probability of not winning the game.

19. There is a 15% chance it will snow and a 15% chance it will rain. What is the probability that it will neither snow nor rain?

20. The odds against winning a contest are 99 : 1. What is the probability of not winning the contest?

21. The odds of choosing a white marble from a bag are 1 : 9. Find the probability of not choosing a white marble.

22. The probability of a spinner landing on green is 25%. What are the odds of the spinner not landing on green?
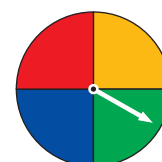
Use the spinner for Exercises 23–28.

23. *P*(red)   24. *P*(green)

25. *P*(not blue)   26. odds in favor of yellow

27. odds against red   28. odds against green

29. **Write About It** A number cube is rolled. Which event has a greater theoretical probability: rolling a number less than 3 or rolling a number greater than three? Explain.

30. **/// ERROR ANALYSIS ///** The odds in favor of an event are 1 : 4. Two students converted these odds into the probability of the event NOT happening. Which is incorrect? Explain the error.

Ⓐ	Ⓑ
Odds in favor of event 1:4	Odds in favor of event 1:4
Probability that event will not happen $\frac{1}{5}$	Probability that event will not happen $\frac{4}{5}$

31. **Critical Thinking** The odds in favor of a certain event are the same as the odds against that event. What is the probability of the event occurring?

32. **MULTI-STEP TAKS PREP** This problem will prepare you for the Multi-Step TAKS Prep on page 744.

Chutes and Ladders is a children's game that uses a spinner with the numbers 1 through 6.

  a. What is the probability of a spinning a 3?

  b. What is the probability of spinning an odd number?

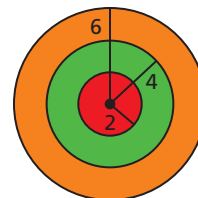 c. What is the probability of spinning a number that is less than or equal to 4?

33. Write About It Explain how to convert odds to probability.

34. Geometry The radius of each circle in the diagram is given. Find the probability that a point chosen at random will lie in the red area of the diagram.

35. Two coins are tossed. What is the probability that at least one of the coins lands with heads showing?

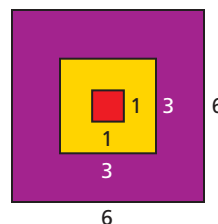   (A) 25%   (B) $33\frac{1}{3}$%   (C) 50%   (D) 75%

36. A standard number cube is rolled. Which has the greatest probability?

    (F) P(even)   (G) P(less than 5)   (H) P(not 2)   (J) P(greater than 3)

37. Find the probability that a point chosen at random would fall in the yellow area.

    (A) $\dfrac{1}{6}$ (C) $\dfrac{4}{9}$

    (B) $\dfrac{2}{9}$ (D) $\dfrac{2}{3}$

CHALLENGE AND EXTEND

Use the results of 3 coin-tossing experiments in the table for Exercise 38.

38. Find the experimental probability for

  a. experiment 1.

  b. experiment 2.

  c. experiment 3.

  d. Find the theoretical probability of heads.

	Experiment		
	1	2	3
Number of Tosses	10	100	1000
Heads	4	48	502

  e. Write About It How do the experimental probabilities of each experiment compare to the theoretical probability?

SPIRAL REVIEW

39. The table shows the volume of water in an office water cooler over time. Find the rate of change for each time period. For which time period did the volume of water decrease at the slowest rate? *(Lesson 5-3)*

Time of day	7:00 A.M.	9:00 A.M.	1:00 P.M.	4:00 P.M.	5:00 P.M.
Volume (gal)	4.2	3.8	2.7	1.2	0.8

Factor each trinomial. *(Lesson 8-4)*

40. $2x^2 + x - 21$ **41.** $4x^2 - 7x + 3$ **42.** $6x^2 + 23x + 20$

An experiment consists of choosing a card out of a deck and recording the results. Use the table to find the experimental probability of each event. *(Lesson 10-5)*

43. choosing a heart

44. choosing a heart or a diamond

45. not choosing a club

Outcome	Frequency
Hearts	2
Diamonds	6
Spades	5
Clubs	7

Independent and Dependent Events

Objectives
Find the probability of independent events.

Find the probability of dependent events.

Vocabulary
independent events
dependent events

Why learn this?
You may need to understand independent and dependent events to determine the number of reading selections available.

Adam's teacher gives the class two lists of titles and asks each student to choose two of them to read. Adam can choose one title from each list or two titles from the same list.

One title from each list		Two titles from the same list	
Animal Farm	Never Cry Wolf	Animal Farm	Animal Farm
Ethan Frome	Night	Ethan Frome	Ethan Frome
Frankenstein	Things Fall Apart	Frankenstein	Frankenstein
Great Expectations	Wish You Well	Great Expectations	Great Expectations
Jane Eyre	Wuthering Heights	Jane Eyre	Jane Eyre

Choosing a title from one list does not affect the number of titles to choose from on the other list. The events are *independent*.

Choosing a title from one of the lists changes the number of titles that can be chosen from the same list. The events are *dependent*.

Events are **independent events** if the occurrence of one event does not affect the probability of the other. Events are **dependent events** if the occurrence of one event does affect the probability of the other.

EXAMPLE 1 **Classifying Events as Independent or Dependent**

Tell whether each set of events is independent or dependent. Explain your answer.

A **A dime lands heads up and a nickel lands heads up.**

The result of tossing a dime does not affect the result of tossing a nickel, so the events are independent.

B **You choose a colored game piece in a board game, and then your sister picks another color.**

Your sister cannot pick the same color you picked, and there are fewer game pieces for your sister to choose from after you choose, so the events are dependent.

CHECK IT OUT!

Tell whether each set of events is independent or dependent. Explain your answer.

1a. A number cube lands showing an odd number. It is rolled a second time and lands showing 6.

1b. One student in your class is chosen for a project. Then another student in the class is chosen.

Independent and Dependent Events

I use the everyday meanings of independent and dependent to remember their mathematical meanings.

Lydia Delgado
Northridge High School

The math class I take next year depends on which math classes I have already taken.	*The history class I take next year is independent from (does not depend on) which math classes I have taken.*
For dependent events, the occurrence of one affects the probability of the other.	*For independent events, the occurrence of one does not affect the occurrence of the other.*

Suppose an experiment involves flipping two fair coins. The sample space of outcomes is shown by the tree diagram. Determine the theoretical probability of both coins landing heads up.

1st Coin   **2nd Coin**

H ———— H
      ———— T

T ———— H
      ———— T

There are four possible outcomes in the sample space: $\{(H, H), (H, T), (T, H), (T, T)\}$

Only one outcome includes both coins landing heads up.

The theoretical probability of both coins landing heads up is $\frac{1}{4}$.
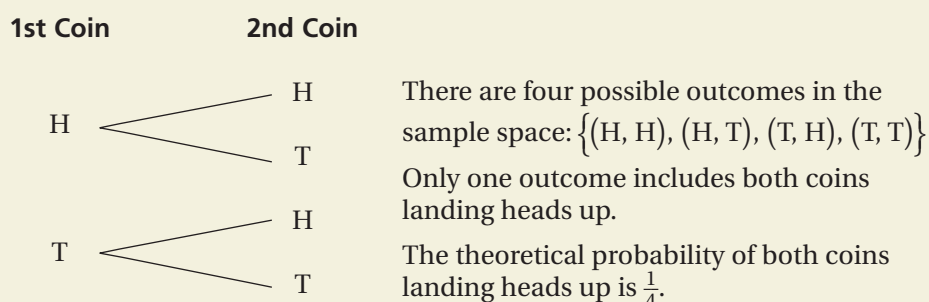
Now look back at the separate theoretical probabilities of each coin landing heads up. The theoretical probability in each case is $\frac{1}{2}$. The product of these two probabilities is $\frac{1}{4}$, the same probability shown by the tree diagram.

To determine the probability of two independent events, multiply the probabilities of the two events.

Know it!
Note

> **Probability of Independent Events**
>
> If A and B are independent events, then $P(A \text{ and } B) = P(A) \cdot P(B)$.

EXAMPLE 2 **Finding the Probability of Independent Events**

A An experiment consists of randomly selecting a marble from a bag, replacing it, and then selecting another marble. The bag contains 7 blue marbles and 3 yellow marbles. What is the probability of selecting a yellow marble and then a blue marble?

Because the first marble is replaced after it is selected, the sample space for each selection is the same. The events are independent.

$P(\text{yellow, blue}) = P(\text{yellow}) \cdot P(\text{blue})$

$= \frac{3}{10} \cdot \frac{7}{10}$ *The probability of selecting yellow is $\frac{3}{10}$, and the probability of selecting blue is $\frac{7}{10}$.*

$= \frac{21}{100}$

B When a person rolls 2 dice and they land showing the same number, we say the person rolled doubles. What is the probability of rolling doubles 3 times in a row?

The result of one roll does not affect any following rolls. The events are independent.

When you roll a pair of dice, there are 36 possible outcomes, six of which are doubles:
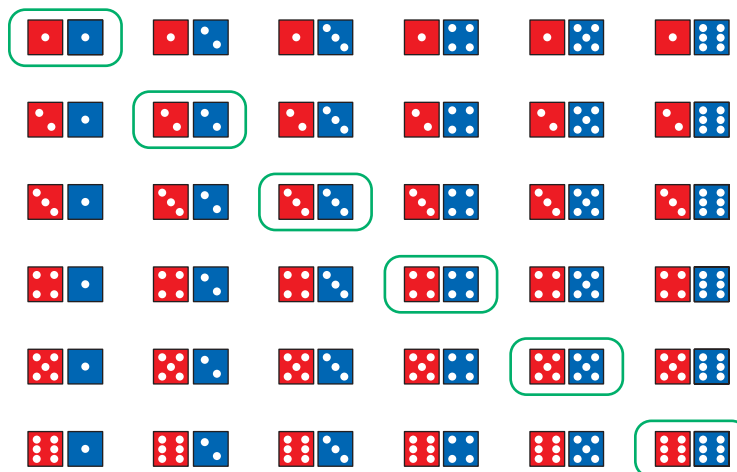
So, the probability of rolling doubles once is $P(\text{double}) = \dfrac{6}{36} = \dfrac{1}{6}$.

$P(\text{double, double, double}) = P(\text{double}) \cdot P(\text{double}) \cdot P(\text{double})$

$$= \frac{1}{6} \cdot \frac{1}{6} \cdot \frac{1}{6}$$

$$= \frac{1}{216}$$

CHECK IT OUT!

2. An experiment consists of spinning the spinner twice. What is the probability of spinning two odd numbers?

Suppose an experiment involves drawing marbles from a bag. Determine the theoretical probability of drawing a red marble and then drawing a second red marble *without replacing the first one*.

1st Draw   **2nd Draw**

The sample space for the second draw is not the same as the sample space for the first draw. There are fewer marbles in the bag for the second draw.

This means the events are dependent.

Probability of drawing a red marble on the first draw $= \dfrac{3}{9} = \dfrac{1}{3}$

Probability of drawing a red marble on the second draw $= \dfrac{2}{8} = \dfrac{1}{4}$

To determine the probability of two dependent events, multiply the probability of the first event times the probability of the second event after the first event has occurred.

Probability of Dependent Events

If A and B are dependent events, then $P(A \text{ and } B) = P(A) \cdot P(B \text{ after } A)$.

EXAMPLE 3

Problem-Solving Application

There are 7 pink flowers and 5 yellow flowers in a bunch. Jane selects a flower at random, and then Leah selects a flower at random from the remaining flowers. What is the probability that Jane selects a pink flower and Leah selects a yellow flower?

1 Understand the Problem

The **answer** will be the probability that a yellow flower is chosen after a pink flower is chosen.
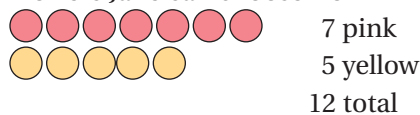
List the **important information:**
- Jane chooses a pink flower from 7 pink flowers and 5 yellow flowers.
- Leah chooses a yellow flower from 6 pink flowers and 5 yellow flowers.
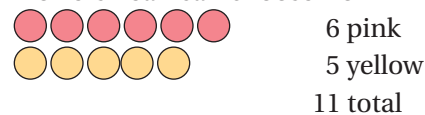
2 Make a Plan

Draw a diagram.

Flowers Jane can choose from

7 pink
5 yellow
12 total

Flowers Leah can choose from

6 pink
5 yellow
11 total

After Jane selects a flower, the sample space changes. So the events are dependent.

3 Solve

$P(\text{pink and yellow}) = P(\text{pink}) \cdot P(\text{yellow after pink})$

$$= \frac{7}{12} \cdot \frac{5}{11}$$

Jane selects one of 7 pink flowers from 12 total flowers. Then Leah selects one of 5 yellow flowers from the 11 flowers left.

$$= \frac{35}{132}$$

The probability that Jane selects a pink flower and Leah selects a yellow flower is $\frac{35}{132}$.

4 Look Back

Drawing a diagram helps you see how the sample space changes. This means the events are dependent, so you can use the formula for probability of dependent events.

3. A bag has 10 red marbles, 12 white marbles, and 8 blue marbles. Two marbles are randomly drawn from the bag. What is the probability of drawing a blue marble and then a red marble?

THINK AND DISCUSS

1. Give an example of two events that are dependent. Explain why the events are dependent.

2. **GET ORGANIZED** Copy and complete the graphic organizer.

	Example	Probability
Dependent Events	A = B =	P(A and B) =
Independent Events	A = B =	P(A and B) =

10-7

Exercises

TAKS Grade 8 Obj. 5, 6
Grades 9–11 Obj. 5, 9, 10

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GUIDED PRACTICE

1. **Vocabulary** Two events are _____?_____ if the occurrence of one event affects the probability of the other event. (*independent* or *dependent*)

Tell whether each set of events is independent or dependent. Explain your answers.

2. You draw a heart from a deck of cards and set it aside. Then you draw a club from the deck of cards.

3. You guess "true" on two true-false questions.

4. Your brother calls you on the phone. You hang up the phone, and then your neighbor calls you.

5. You order from a menu, and then your friend orders a different meal.

6. A doctors' office schedules several patients. Then you make an appointment.

7. A coin is tossed three times. What is the probability of the coin landing heads up three times?

8. Seven cards are numbered from 1 to 7 and placed in a box. One card is selected at random and replaced. Another card is randomly selected. What is the probability of selecting two odd numbers?

9. Stacey rolls two number cubes. What is the probability that the sum of the numbers on the two number cubes is 7?

10. A number cube is rolled twice and a coin is tossed once. What is the probability of the coin landing heads up and the number cube landing with 2 showing both times?

11. A spinner with four equal sections of red, yellow, green, and blue is spun twice. What is the probability that it lands on yellow and then on green?

12. A bag contains 4 red marbles, 3 white marbles, and 6 blue marbles. What is the probability of randomly selecting a red marble, setting it aside, and then randomly selecting a white marble from the bag?

13. Seven cards are numbered from 1 to 7 and placed in a box. One card is selected at random and not replaced. Another card is randomly selected. What is the probability of selecting two odd numbers?

14. There are 15 boys and 14 girls in a room. Two of them are selected at random to take a survey. What is the probability that the two people selected will be girls?

PRACTICE AND PROBLEM SOLVING

Tell whether each set of events is independent or dependent. Explain your answer.

15. The teacher randomly selects two students from the class.

16. You roll a 3 on a number cube and choose a 3 from a deck of cards.

17. A number cube is rolled three times. What is the probability of rolling three even numbers?

18. Ten cards are numbered from 1 to 10 and placed in a box. One card is selected at random and replaced. Another card is randomly selected. What is the probability of selecting two even numbers?

19. Stacey rolls a number cube and flips a coin. What is the probability that she rolls a 5 and the coin lands heads up?

20. A bag contains 5 red marbles, 3 white marbles, and 4 blue marbles. What is the probability of randomly selecting a red marble, setting it aside, and then randomly selecting another red marble from the bag?

21. Ten cards are numbered from 1 to 10 and placed in a box. One card is selected at random and not replaced. Another card is randomly selected. What is the probability of selecting two even numbers?

22. A game has 6 colored playing pieces. They are red, yellow, green, blue, purple, and white. You and a friend pick your game piece without looking. What is the probability that your friend picks the blue piece and you pick the yellow piece?

TEXAS LINK

School

On the TAKS test, there is no penalty for a wrong answer. It is better to guess than to leave the answer blank, especially if some choices can be eliminated.

23. **School** On a multiple-choice test, each question has 4 possible answers. A student does not know the answers to three questions, so the student guesses.
   a. What is the probability that the student gets all three questions wrong?
   b. What is the probability that the student gets all three questions right?

Tell whether each set of events is independent or dependent. Explain your answer.

24. Pick "Joe" from a box of names, replace it, and then pick "Craig."

25. Pick "Joe" from a box of names, set it aside, and then pick "Craig."

26. Roll a prime number on a number cube and get tails when flipping a coin.

27. Roll an even number, then an odd number, and then a 1 on a number cube.

28. This problem will prepare you for the Multi-Step TAKS Prep on page 744.
Yahtzee is a game that involves rolling five dice. On his or her turn, a player can roll up to three times to try to score points in various categories. Rolling a "Yahtzee" means rolling five of a kind, or five of the same number.

a. Juan has rolled twice and has three 5's showing. He rolls the remaining two dice. What is the probability that both dice will land showing 5?

b. Shauna has two 3's showing. She has one more roll with the remaining three dice. What is the probability that all three dice will land showing 3?

c. Mike rolls all five number cubes and all of them land showing 6. What is the probability of getting five 6's in one roll?

29. A bag contains 3 red, 5 blue, and 2 white marbles.

a. Find the probability of randomly picking a red marble, replacing it, and then picking a blue marble.

b. Find the probability of randomly picking a red marble, setting it aside, and then picking a blue marble.

c. Find the probability of randomly picking a red marble, replacing it, and then picking another red marble.

d. Find the probability of randomly picking a red marble, setting it aside, and then picking another red marble.

30. Entertainment Joe and Maria are playing a board game. On each turn, the player rolls two number cubes. Both players have two turns remaining.

a. Joe will win if he rolls double 6's on both turns. What is the probability that Joe will roll double 6's on both turns?

b. Maria will win if she rolls 2 on the first turn and 12 on the second turn. What is the probability that Maria will roll 2 on the first turn and 12 on the second turn?

c. Write About It Who has the better probability of winning? Explain.

31. Tamika has $2.50 in quarters in her pocket, including four state quarters. She reaches into her pocket and takes out two quarters, one at a time. What is the probability that they are both state quarters?

32. Ten cards are numbered 1 through 10 and placed in a bag. You draw a card, set it aside, and draw another card. What is the probability that you will draw two numbers that are divisible by 3?

33. Critical Thinking What is the probability of a coin landing heads up on two flips if it lands tails up on the first flip? Explain.

34. Write About It Explain what it means for two events to be independent.

TEST PREP

35. A number cube is rolled twice. What is the probability of getting a 2 on both rolls?

Ⓐ $\frac{1}{3}$　　　Ⓑ $\frac{1}{4}$　　　Ⓒ $\frac{1}{9}$　　　Ⓓ $\frac{1}{36}$

36. In baseball, Julio averages 3 hits in every 10 at bats. What is the probability that he will get hits in both of his next two at bats?

Ⓕ 0.03　　　Ⓖ 0.09　　　Ⓗ 0.3　　　Ⓙ 0.9

37. Two people from a group of 30 will be selected at random for a prize. Twenty people in the group are women. What is the probability that both people selected will be men?

Ⓐ $\dfrac{3}{29}$　　　　Ⓑ $\dfrac{38}{87}$　　　　Ⓒ $\dfrac{56}{87}$　　　　Ⓓ $\dfrac{1}{10}$

38. Ravi has 10 pairs of socks in a drawer, but none of the pairs are matched up. Each pair is a different color, and one pair is blue. Ravi has to pick his socks in the dark so he does not wake his brother. Which expression can be used to find the probability that Ravi will choose a blue sock and then the matching sock?

Ⓕ $\dfrac{1}{20} + \dfrac{1}{20}$　　　Ⓖ $\dfrac{1}{10} \cdot \dfrac{1}{20}$　　　Ⓗ $\dfrac{1}{20} + \dfrac{1}{19}$　　　Ⓙ $\dfrac{1}{10} \cdot \dfrac{1}{19}$

CHALLENGE AND EXTEND

39. **Basketball** Terrance has made 90% of the free throws he has attempted at basketball practice. What is the probability that he will make the next three free throws he attempts?

40. A number cube is rolled three times. What is the probability of rolling a 5 at least once?

Geometry Use the grid for Exercises 41–44.

On the grid at right, one small square represents 1% probability.

The pink area represents the probability that event *A* occurs.

The blue area represents the probability that event *B* occurs.

The area where the two colors overlap represents the probability that both events occur.

Use the grid to find each probability.

41. Event *A* occurs.

42. Event *B* occurs.

43. Event *A* occurs AND event *B* occurs.

44. Neither event *A* nor event *B* occurs.
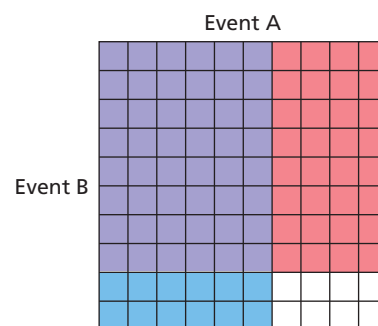
SPIRAL REVIEW

45. A tennis player serves 2 aces (unreturned serves) for every 17 serves. If the player serves 204 times in the next match, how many serves would you expect to be aces? *(Lesson 2-6)*

Compare the graph of each function to the graph of $f(x) = x^2$. *(Lesson 9-4)*

46. $g(x) = 4x^2 - 1$　　　　**47.** $g(x) = \dfrac{1}{5}x^2$　　　　**48.** $g(x) = -x^2 + 3$

Find the theoretical probability of each outcome. *(Lesson 10-6)*

49. Randomly selecting a blue marble out of a bag with 6 red and 9 blue marbles

50. Rolling a number less than 10 on a number cube

51. Randomly selecting A, E, I, O, or U from all letters of the alphabet

Compound Events

When two events cannot happen at the same time, they are called *mutually exclusive* events. When two events can happen at the same time, they are called *inclusive* events. You can use sample spaces to determine the probabilities of mutually exclusive events and inclusive events.

When finding the probablity of a compound event, you should first determine whether the simple events involved are dependent or independent.

Activity 1

Suppose you are playing Monopoly and are "just visiting" the Jail. If you roll 7 on your next turn, you will land on Community Chest. If you roll 12, you will land on Chance. What is the probability that on your next roll you land on either Community Chest or Chance?

You cannot land on Community Chest and Chance at the same time, so the events are mutually exclusive. You cannot roll 7 and 12 at the same time, so those events are also mutually exclusive.

The table shows some of the totals when rolling two fair dice. Copy and complete the table. Use one color to circle all rolls with a total of 7. Use a second color to circle all rolls with a total of 12.

+	1	2	3	4	5	6
1	2	3	4	5	▪	▪
2	3	4	▪	▪	▪	▪
3	4	5	▪	▪	▪	▪
4	5	▪	▪	▪	▪	▪
5	6	▪	▪	▪	▪	▪
6	7	▪	▪	▪	▪	▪

Try This

1. What is the total number of possible rolls?

2. What is the probability that the total will be 7?

3. What is the probability that the total will be 12?

4. What is the probability that the total will be 7 or 12?

5. What do you notice about the probabilities in Problems 2, 3, and 4?

Suppose you are three spaces away from Community Chest and eight spaces away from Chance.

6. What is the probability that on your next roll the total will be 3?

7. What is the probability that on your next roll the total will be 8?

8. What is the probability that on your next roll the total will be 3 or 8?

9. Complete the following statement:

   The probability that one of two mutually exclusive events will occur is the
   ___?___ of the probabilities of the individual events.

In this lab, you discovered this rule: If A and B are mutually exclusive events, then $P(A \text{ or } B) = P(A) + P(B)$.

Activity 2

Suppose you are playing Yahtzee and on the last roll of your turn, you roll two of the five dice. You need to roll either a 1 or a 5 to make what is called a "small straight" (four consecutive numbers).

What is the probability that you will make a small straight?

You can roll a 1 and a 5 at the same time on different dice. These events are inclusive.

(If you roll a 1 and a 5, you will need to look at only four of the dice to make a small straight.)

There are three outcomes that involve rolling a 1 or rolling a 5.

Case 1	Case 2	Case 3
At least one 1	At least one 5	Both a 1 and a 5

Try This

10. Find the probability of Case 1, rolling at least one 1.

    a. What is the probability of rolling ⚀ on the first die and any number on the second die?

    b. What is the probability of rolling any number on the first die and ⚀ on the second die?

    c. What is the probability of rolling ⚀ on the first die and ⚀ on the second die?

    d. Parts **a** and **b** both include rolling ⚀ on the first die and ⚀ on the second die. You need to count this outcome only once, so subtract the probability from part **c** from the sum of parts **a** and **b.**

11. Find the probability of Case 2, rolling at least one 5. (*Hint:* Repeat the steps in Problem 10.)

12. Find the probability of Case 3, rolling a 1 and a 5.

13. The probability of rolling a small straight, is the probability of rolling ⚀ *or* ⚄.

Subtract the probability of rolling both ⚀ *and* ⚄ from the sum of the probabilities of rolling ⚀ on one of the dice (Problem 10) and rolling ⚄ on one of the dice (Problem 11).

14. Complete the following statement:

The probability that one of two inclusive events will occur is the ___?___ of the probabilities of the individual events ___?___ the probability of the intersection of the two events.
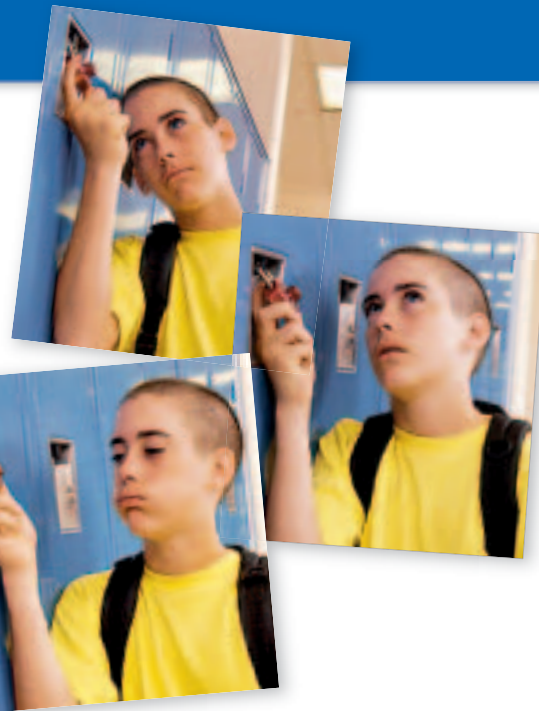
In this lab, you discovered this rule:

If A and B are inclusive events, then $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$.

10-8 Combinations and Permutations

Who uses this?
Security companies use permutations to create many security codes using a limited number of digits or letters. (See Exercise 8.)

Sometimes there are too many possible outcomes to make a tree diagram or a list. The *Fundamental Counting Principle* is one method of finding the number of possible outcomes.

> **Know it! Note**

Fundamental Counting Principle

If there are *m* ways to choose a first item and *n* ways to choose a second item after the first item has been chosen, then there are *m · n* ways to choose both items.

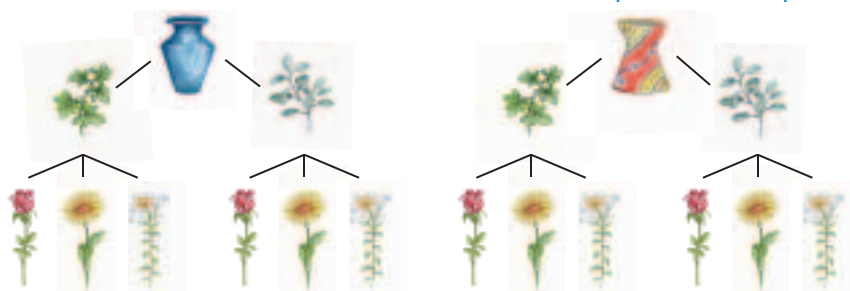The Fundamental Counting Principle can also be used when there are more than two items to choose.

EXAMPLE 1 **Using the Fundamental Counting Principle**

A florist is arranging centerpieces that include 1 flower, 1 plant, and 1 vase. The florist has 2 kinds of vases, 2 kinds of plants, and 3 kinds of flowers to choose from. How many different centerpieces are possible?

Method 1 Use a tree diagram. *Follow each branch of the tree diagram to find all of the possible centerpieces.*

There are 12 possible centerpieces.

Method 2 Use the Fundamental Counting Principle.

2 · 2 · 3   *There are 2 choices for the first item, 2 choices for the*
   12   *second item, and 3 choices for the third item.*

There are 12 possible centerpieces.

CHECK IT OUT!

1. A voicemail system password is 1 letter followed by a 3-digit number less than 600. How many different voicemail passwords are possible?

A **compound event** consists of two or more simple events, such as a rolled number cube landing with 3 showing and a tossed coin landing heads up. (A simple event has only one outcome, such as rolling a 3 on a number cube.) For some compound events, the order in which the simple events occur is important.

A **combination** is a grouping of outcomes in which the order does not matter. A **permutation** is an arrangement of outcomes in which the order does matter.

EXAMPLE 2   **Finding Combinations and Permutations**

Tell whether each situation involves combinations or permutations. Then give the number of possible outcomes.

A A street vendor sells cashews, peanuts, and almonds. How many different ways are there to mix two kinds of nuts?

List all of the possible groupings:

cashews and peanuts	peanuts and almonds	almonds and cashews
peanuts and cashews	almonds and peanuts	cashews and almonds

The order of the outcomes is not important, so this situation involves combinations. Eliminate the groupings that are duplicates.

cashews and peanuts	peanuts and almonds	almonds and cashews
~~peanuts and cashews~~	~~almonds and peanuts~~	~~cashews and almonds~~

There are 3 different ways to mix two kinds of nuts.

B Karen is painting her bedroom. She has orange, green, blue, and purple paint. She plans to use one color as a base coat and stencil a design with another color. How many different ways can she do this?

List all of the possible groupings:

The order of the outcomes is important, because one color is painted on top of the other. This situation involves permutations.

There are 12 different ways for Karen to paint her room.

CHECK IT OUT! Tell whether each situation involves combinations or permutations. Then give the number of possible outcomes.

2a. Ingrid is stringing three different types of beads on a bracelet. How many ways can she use one bead of each type to string the next three beads?

2b. Nathan wants to order a sandwich with two of the following ingredients: mushroom, eggplant, tomato, and avocado. How many different sandwiches can Nathan choose?

The factorial of a number is the product of the number and all the natural numbers less than the number. The factorial of 5 is written 5! and is read "five factorial." $5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$. Factorials can be used to find the number of combinations and permutations that can be made from a set of choices.

Suppose you want to make a five-letter password from the letters *A, B, C, D,* and *E* without repeating a letter. You have 5 choices for the first letter, but only 4 choices for the second letter. You have one fewer choice for each letter of the password.

First letter   Second letter   Third letter   Fourth letter   Fifth letter

5 choices ✕ 4 choices ✕ 3 choices ✕ 2 choices ✕ 1 choice = 120 permutations

Suppose you want to make a three-letter password from the 5 letters *A, B, C, D,* and *E* without repeating a letter. Again, you have one fewer choice for each letter of the password.

First letter   Second letter   Third letter

There are 5 choices (A, B, C, D, E), and you are choosing 3 of them.

5 choices ✕ 4 choices ✕ 3 choices = 60 permutations

The number of permutations is $\dfrac{5!}{2!}$, or $\dfrac{5 \cdot 4 \cdot 3 \cdot \cancel{2} \cdot \cancel{1}}{\cancel{2} \cdot \cancel{1}} = 5 \cdot 4 \cdot 3 = 60$.

Permutations

FORMULA	The number of permutations of n things chosen r at a time:	$_nP_r = \dfrac{n!}{(n-r)!}$.
EXAMPLE	A club will choose a president, a vice president, and a secretary from a list of 8 people. How many ways can the club choose the 3 officers? The position that each person takes matters, so this situation involves permutations. Think: There are 8 people, and the club will choose 3 of them.	$_8P_3 = \dfrac{8!}{(8-3)!} = \dfrac{8!}{5!} = 336$

E X A M P L E   3   Finding Permutations

Lee brings 7 CDs numbered 1–7 to a party. How many different ways can he choose the first 4 CDs to play?

The order in which the CDs are played matters, so use the formula for permutations

$_7P_4 = \dfrac{7!}{(7-4)!} = \dfrac{7!}{3!}$ $n = 7$ and $r = 4$.

$\qquad = \dfrac{7 \cdot 6 \cdot 5 \cdot 4 \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}}{\cancel{3} \cdot \cancel{2} \cdot \cancel{1}}$ *A number divided by itself is 1, so you can divide out common factors in the numerator and denominator.*

$\qquad = 7 \cdot 6 \cdot 5 \cdot 4 = 840$

There are 840 different ways Lee can choose to play the 4 CDs.

CHECK IT OUT!   **3.** How many different ways can 9 people line up for a picture?

The formula for combinations also involves factorials.

Combinations

FORMULA	The number of combinations of n things chosen r at a time:	$_nC_r = \dfrac{n!}{r!(n-r)!}$.
EXAMPLE	A club will form a 3-person committee from a list of 8 people. How many ways can the club choose the 3 people? The position that each person takes does not matter, so this situation involves combinations. Think: There are 8 people, and the club will choose 3 of them.	$_8C_3 = \dfrac{8!}{3!(8-3)!} = \dfrac{8!}{3!5!} = 56$

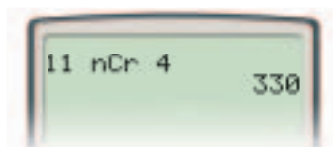**E X A M P L E   4**   **Finding Combinations**

There are 11 different cereals for sale at a grocery store. How many different ways can a shopper select 4 different cereals?

The order in which the cereals are selected does not matter, so use the formula for combinations.

Method 1 Use the formula for combinations.

$$_{11}C_4 = \frac{11!}{4!\,(11-4)!} = \frac{11!}{4!7!} \qquad \textcolor{blue}{n = 11 \text{ and } r = 4.}$$

$$= \frac{11 \cdot 10 \cdot 9 \cdot 8 \cdot \cancel{7} \cdot \cancel{6} \cdot \cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}}{(4 \cdot 3 \cdot 2 \cdot 1)(\cancel{7} \cdot \cancel{6} \cdot \cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1})}$$

$$= \frac{7920}{24} = 330$$

Method 2 Use the **nCr** function of a calculator.

```
11 nCr 4
           330
```

Helpful Hint

You can also use a calculator to find permutations. Look for the **nPr** function on your calculator.

There are 330 different ways the shopper can choose 4 different cereals.

CHECK IT OUT!

4. A basketball team has 12 members who can play any position. How many different ways can the coach choose 5 starting players?

THINK AND DISCUSS

1. Explain how to find $\frac{10!}{9!}$ without a calculator.

2. GET ORGANIZED Copy and complete the graphic organizer.

	Fundamental Counting Principle	Permutation	Combination
When to Use			
Formula			

GUIDED PRACTICE

1. **Vocabulary** A _____?_____ is an arrangement of outcomes in which the order does not matter. (*compound event, permutation*, or *combination*)

SEE EXAMPLE **1**
p. 736

2. The menu for a restaurant is shown. How many different meals with one salad, one soup, one entree, and one dessert are possible?

3. You have 4 colors of wrapping paper and 2 kinds of ribbon. How many different ways are there to use one color of paper and one type of ribbon to wrap a package?

Create a Lunch
Choose one from each group $5.00

Soup
• Minestrone
• Vegetable Beef
• Chicken Noodle

Salad
• Garden Salad
• Caesar Salad

Entrée
• Grilled Chicken
• Baked Fish
• Spaghetti

Dessert
• Sorbet
• Fruit Salad
• Frozen Yogurt
• Blended Yogurt w/ Fruit

SEE EXAMPLE **2**
p. 737

Tell whether each situation involves combinations or permutations. Then give the number of possible outcomes.

4. How many different ways can 4 people be seated in a row of 4 seats?

5. How many different kinds of punch can be made from 2 of the following: cranberry juice, apple juice, orange juice, and grape juice?

SEE EXAMPLE **3**
p. 738

6. An airport identification code is made up of three letters, like the examples shown. How many different airport identification codes are possible? (Assume any letter can be repeated and can appear in any position.)

7. A manager is scheduling interviews for job applicants. She has 8 time slots and 6 applicants to interview. How many different interview schedules are possible?

8. An e-mail server password must be 6 different lowercase letters. How many different e-mail server passwords are possible?

AMA Amarillo International Airport, Amarillo, Texas

DFW Dallas/Fort Worth International Airport, Texas

ELP El Paso International Airport, El Paso, Texas

HRL Rio Grande Valley International Airport, Harlingen, Texas

SAT San Antonio International Airport, San Antonio, Texas

SEE EXAMPLE **4**
p. 739

9. Francine is deciding which after-school clubs to join. She has time to participate in 3. How many different ways can Francine choose which 3 clubs to join?

10. Laura is making gift baskets. Each basket has 2 gifts, and she has 5 gifts to choose from. How many different gift baskets can Laura make?

11. How many different ways can a contest judge select 5 finalists from 20 contestants?

Join a Club!
Science Club   Math Club
Drama Club   Dance Club
Chess Club   Spanish Club
Student Government

PRACTICE AND PROBLEM SOLVING

12. Maria looks in her closet and exclaims, "I have nothing to wear!" How many different outfits of one shirt, one pair of shorts, and one pair of sandals are possible using the items shown?

13. When a customer buys tickets for a concert, the ticket office assigns a confirmation code that is made up of 2 lowercase consonants, followed by 3 numbers. A letter or number may be repeated. How many different confirmation codes are possible?

Tell whether each situation involves a combination or a permutation. Then give the number of possible outcomes.

14. A team of archeologists divides a dig site into 3 areas. They dig one area at a time. How many different ways can they order the 3 areas?

15. To decide which team will lead the class discussion, a teacher writes the names of 5 students on slips of paper and puts them in a hat. Then she draws 2 names. How many teams of two are possible?

16. The code for a bicycle lock is made up of 4 digits from 0 through 9. How many different codes are possible?

17. A television station has 5 different commercials to play during the news. In how many different ways can they order the commercials?

18. David's summer reading list has 9 books. How many different ways can David select 3 books to read?

19. Steve draws a hand of 7 cards from a deck of 52 different cards. How many different hands are possible?

20. History The North American Numbering Plan (NANP) was first used in 1947. It is a system for assigning telephone numbers and area codes.

   a. Originally, the NANP allowed only 3-digit area codes whose first digit was not 0 or 1, and whose second digit was always 0 or 1. How many different area codes were possible under this system?

   b. In 1995, because of increased demand, the NANP removed the restriction that the second digit of the area code must be 0 or 1. How many more area codes did this make possible?

21. Critical Thinking For $n = 6$ and $r = 2$, which is larger: $_nP_r$ or $_nC_r$? Explain why this is true.

22. Write About It Brian forgot the combination to open his locker. Explain why a "combination lock" should be called a "permutation lock."

23. Write About It You roll a number cube 6 times. Explain how to determine the number of possible outcomes. Would a tree diagram be useful in solving this problem? Explain why or why not.

24. This problem will prepare you for the Multi-Step TAKS Prep on page 744.

Many states have lotteries to raise money. In one lottery, the players choose 6 different numbers from the numbers 1–49. The order of the numbers does not matter.

a. How many combinations are there?

b. The probability that your first number is on the winning ticket is $\frac{6}{49}$. The probability that your second number matches is $\frac{5}{48}$. The probabilities of the third, fourth, fifth, and sixth numbers matching are $\frac{4}{47}$, $\frac{3}{46}$, $\frac{2}{45}$, and $\frac{1}{44}$. The product of these probabilities is the probability of winning the lottery with one ticket. Find that probability. How does it relate to the number of combinations shown in part **a?**

25. Technology Nancy's password for her online banking program is made up of 6 letters. It does not matter whether the letters are uppercase or lowercase and any letter may be repeated.

a. What is the probability of someone correctly guessing Nancy's password?

b. How many hours would it take for someone to try all the possible passwords if the person could guess one password each second?

c. Write About It Explain why you think some Internet companies require that passwords be at least 6 characters long.

TEST PREP

26. Which situation is best modeled by the expression $_{10}C_4$?

Ⓐ Lisa's salon offers 10 kinds of shampoo and 4 kinds of conditioner. How many different ways can she shampoo and condition a client's hair?

Ⓑ Sara has a box of 10 colored pencils. How many different ways can she choose 4 colored pencils from the box?

Ⓒ A personal identification number (PIN) is a 4-digit number. Digits can be repeated in a PIN. How many different PINs are possible?

Ⓓ There are 10 chairs placed around a table. How many different ways can 4 people seat themselves around the table?

27. Which of the following is equal to 4!?

Ⓕ 10! − 6!　　Ⓖ $\frac{8!}{2!}$　　Ⓗ (2!)(2!)　　Ⓙ $\frac{24!}{23!}$

28. Which table represents the values of $_xC_4$ for the given values of x?

Ⓐ

x	$_xC_4$
5	5
6	30
7	210
8	1680

Ⓑ

x	$_xC_4$
5	120
6	360
7	840
8	1680

Ⓒ

x	$_xC_4$
5	5
6	15
7	35
8	70

Ⓓ

x	$_xC_4$
5	30
6	90
7	210
8	240

29. Short Response The periodic table of the elements is a chart listing 112 different elements. Maria must give a presentation on 3 different elements from the periodic table. How many ways can she choose which elements to present? Show your work and explain each step.

CHALLENGE AND EXTEND

Simplify each expression.

30. $\dfrac{8!}{3!}$

31. $\dfrac{7!4!}{6!}$

32. $\dfrac{_7P_5}{_7P_3}$

33. The chart shows the different toppings offered by a pizza shop.

    a. How many different pizzas with 2 toppings are possible?

    b. How many different pizzas with 6 toppings are possible?

    c. What do you notice about your answers to parts **a** and **b**? Explain why you think this is.

Pizza Ingredients
- Extra Cheese
- Mushrooms
- Sausage
- Pineapple
- Green Peppers
- Pepperoni
- Onions
- Green Olives

34. Use the permutation and combination formulas you learned in this lesson.

    a. What is the number of permutations when $n = r$?

    b. What is the number of combinations when $n = r$?

35. You roll a number cube six times in a row. What is the probability that you roll the numbers 1 through 6 in order?

SPIRAL REVIEW

Simplify each expression. *(Lesson 1-7)*

36. $4.9m + 3.8 - 2.1m - 3.9$ **37.** $4(2x - 1) + 9 - 3x$ **38.** $3a^2 - 5a - 7a^2$

Identify the independent and dependent variables. Write a rule in function notation for each situation. *(Lesson 4-3)*

39. A bathtub fills at a rate of 15 gallons per minute.

40. About 5 seeds should be planted per square inch.

41. A snow removal company charges a contract fee of $300 plus $80 per hour.

42. Five people responded that they spend the following amounts for haircuts during one year: $150, $120, $135, $0, and $145. A researcher concluded that "people spend an average of $110 per year on haircuts." Why is this statistic misleading? *(Lesson 10-4)*

Career Path

Q: What math classes did you take in high school?
A: Algebra, Geometry, Precalculus, Calculus

Q: What math classes have you taken in college?
A: Statistics, Linear Algebra, Problem Solving

Q: What do Biostatisticians do?
A: Biostatisticians apply statistical methods to research scientific questions relating to health and medicine.

Q: What are your plans for the future?
A: After graduating, I would like to get a job at a pharmaceutical company testing the safety and effectiveness of new medicines.

Erika Sheehan
Biostatistics major

MULTI-STEP TAKS PREP

Probability

The Games People Play Probability is involved in most board and card games.

1. There are 108 cards in a deck for the game Uno. The table shows the number of each type of card found in an Uno deck. What is the probability of randomly selecting a red card from a complete deck of Uno cards?

Type of Card	Number in Deck
Blue cards, 0–9	19
Green cards, 0–9	19
Red cards, 0–9	19
Yellow cards, 0–9	19
Draw Two	8 (2 in each color)
Reverse	8 (2 in each color)
Skip	8 (2 in each color)
Wild	4
Wild Draw Four	4

2. Robert is playing Monopoly. He starts the game on the GO space. He wants to avoid the Income Tax space, which he will land on if he rolls a 4 with the two dice. What is the probability that he will land on the Income Tax space?

3. In the game Battleship, each player chooses locations for small ships on a 10-by-10 grid. Each player guesses grid spaces to try to locate the other player's ships. A correct guess equals a hit, and by hitting each space occupied by a ship, the ship is sunk. The battleship piece occupies four grid spaces. What is the probability that a player randomly guesses a correct grid space and hits the other player's battleship?

4. In the game Set, each card can be characterized in four ways. Cards are either red, green, or purple. The design on each card is either an oval, a squiggle, or a diamond. There are either one, two, or three of the design on the card. The designs are either solid, empty, or striped. How many different cards are possible?

Quiz for Lessons 10-5 Through 10-8

10-5 Experimental Probability

An experiment consists of pushing the random select button on a CD player for a disc with five tracks. Use the results in the table to find the experimental probability of each event.

Outcome	Frequency
Track 1	4
Track 2	6
Track 3	2
Track 4	5
Track 5	3

1. Selecting track 4

2. Selecting track 3

3. Not selecting track 2

4. Selecting an odd numbered track

5. Selecting one of the first 3 tracks

6. Ms. Bleakman checks 32 papers and finds 2 with no name. What is the experimental probability that a paper chosen at random will have no name? Ms. Bleakman has the papers of 176 students. Predict the number of papers she will find with no name.

10-6 Theoretical Probability

Find the theoretical probability of each outcome.

7. Landing on an odd number when spinning a spinner with 4 equal spaces marked 1, 4, 6, 8

8. Randomly selecting a vowel from all the letters in the word VOWELS

9. Picking a red marble out of a bag with 3 green, 5 blue, 2 red, and 6 yellow marbles

10. The probability of winning a certain prize is 5%. What are the odds of winning the prize?

11. The odds of choosing a winning ticket are 2:8. What is the probability of choosing a winning ticket?

12. The probability of snow is 70%. What are the odds of it not snowing?

10-7 Independent and Dependent Events

13. A physical education class has 12 boys and 18 girls. Each day, the teacher randomly selects a team captain. Assume that no student is absent. What is the probability that the team captain is a girl two days in a row?

14. After reading to a kindergarten class at the library, Tobey gives out stickers. He has 9 zoo animal stickers and 16 scratch-n-sniff stickers. If Tobey gives the stickers out at random, what is the probability that the first child gets a scratch-n-sniff sticker and the second child gets a zoo animal sticker?

10-8 Combinations and Permutations

15. Luisa is choosing a line-up of CDs to play from beginning to end at her party. She has 12 CDs from which to choose and time to play only 5. How many ways are there for Luisa to arrange the musical line-up?

16. Gavin is shopping for school clothes with his father. After trying on several shirts, Gavin has identified 8 shirts that he likes. His father says Gavin must narrow it down to 5 shirts. How many ways can Gavin choose 5 shirts from the 8 he likes?

$$\begin{bmatrix} 1 & 8 \\ 6 & -1 \\ 5 & 0 \end{bmatrix}$$ } 3 rows

2 columns

EXTENSION Matrices

Objectives
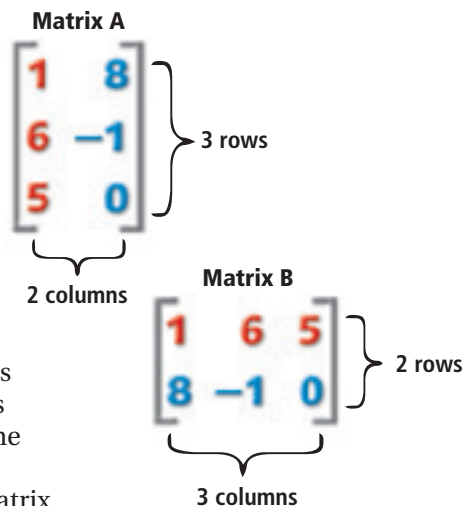Organize data in a matrix.

Add and subtract matrices and multiply a matrix by a scalar.

Vocabulary
matrix
entry
scalar

A **matrix** is a rectangular arrangement of numbers in rows and columns. Each number in the matrix is called an **entry**.

Matrices are described by the number of rows and columns in the matrix. Matrix A is called a 3 × 2 (3 by 2) matrix because it has 3 rows and 2 columns. Matrix B contains the same data, but it is arranged in 2 rows and 3 columns, so it is called a 2 × 3 (2 by 3) matrix.

Matrix B

$$\begin{bmatrix} 1 & 6 & 5 \\ 8 & -1 & 0 \end{bmatrix}$$ } 2 rows

3 columns

EXAMPLE **1** **Organizing Data in a Matrix**

The table shows the number of members in each section of two marching bands. Arrange the data in a matrix. Give the dimensions of the matrix.

Table

	Percussion	Brass	Woodwinds
Northmount High	14	23	16
Western High	19	36	41

Matrix

      P   B   W

North $\begin{bmatrix} 14 & 23 & 16 \\ 19 & 36 & 41 \end{bmatrix}$
West

The matrix has 2 rows and 3 columns, so its dimensions are 2 × 3.

CHECK IT OUT!

1. Use the information above to make a 3 × 2 matrix.

If two matrices have the same dimensions, they can be added or subtracted by finding the sum or difference of the corresponding entries.

EXAMPLE **2** **Adding and Subtracting Matrices**

The matrices show the enrollment by grade level at two high schools during the same year.

A Add the two matrices to find the total enrollment of girls in the tenth grade.

Washington
    Boys Girls

10th $\begin{bmatrix} 141 & 148 \\ 132 & 115 \\ 105 & 128 \end{bmatrix}$
11th
12th

Central
    Boys Girls

10th $\begin{bmatrix} 139 & 127 \\ 141 & 136 \\ 112 & 98 \end{bmatrix}$
11th
12th

$$\begin{bmatrix} 141 & 148 \\ 132 & 115 \\ 105 & 128 \end{bmatrix} + \begin{bmatrix} 139 & 127 \\ 141 & 136 \\ 112 & 98 \end{bmatrix} = \begin{bmatrix} 141+139 & 148+127 \\ 132+141 & 115+136 \\ 105+112 & 128+98 \end{bmatrix} =$$

Total
    Boys Girls

10th $\begin{bmatrix} 280 & 275 \\ 273 & 251 \\ 217 & 226 \end{bmatrix}$
11th
12th

The total enrollment of girls in the tenth grade was 275.

The matrices show the enrollment by grade level at two high schools during the same year.

B Subtract the two matrices to find how many more eleventh-grade boys were enrolled at Washington High than at Central High.

Washington
Boys Girls

	Boys	Girls
10th	141	148
11th	141	136
12th	112	128

Central
Boys Girls

	Boys	Girls
10th	139	127
11th	132	115
12th	105	98

$$\begin{bmatrix} 141 & 148 \\ \mathbf{141} & 136 \\ 112 & 128 \end{bmatrix} - \begin{bmatrix} 139 & 127 \\ \mathbf{132} & 115 \\ 105 & 98 \end{bmatrix} = \begin{bmatrix} 141-139 & 148-127 \\ \mathbf{141-132} & 136-115 \\ 112-105 & 128-98 \end{bmatrix} = $$

Total
Boys Girls

	Boys	Girls
10th	2	21
11th	**9**	21
12th	7	30

There were 9 more eleventh-grade boys enrolled at Washington High than at Central High.

CHECK IT OUT! Use the information above to answer the following.

2a. What was the total enrollment of boys in the twelfth grade that year?

2b. How many more tenth-grade girls were enrolled at Washington High than at Central High that year?

To multiply a matrix by a number, multiply each entry in the matrix by the number. This kind of multiplication is called *scalar* multiplication. The **scalar** is the number by which the matrix is multiplied.

EXAMPLE 3 **Scalar Multiplication of a Matrix**

The matrix shows the number of field goals (FG) and touchdowns (TD) scored by a football team in each quarter of last week's game. This week they scored twice as many field goals and touchdowns in each quarter of the game. Multiply the matrix by 2 to show the number of scores from this week's game.

Points per Quarter

	1st	2nd	3rd	4th
FG	1	0	2	1
TD	0	0	1	1

$$2 \begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} \mathbf{2}(1) & \mathbf{2}(0) & \mathbf{2}(2) & \mathbf{2}(1) \\ \mathbf{2}(0) & \mathbf{2}(0) & \mathbf{2}(1) & \mathbf{2}(1) \end{bmatrix}$$

Multiply each entry in the matrix by the scalar 2.

Points per Quarter

	1st	2nd	3rd	4th
FG	2	0	4	2
TD	0	0	2	2

CHECK IT OUT!

3. The matrix shows the number of short sleeve and long sleeve T-shirts a store has in its inventory. A second store has 3 times as many shirts in inventory. How many of each shirt and size does the second store have in its inventory?

Inventory

	Short sleeve	Long sleeve
Small	42	52
Medium	76	43
Large	19	26

Arrange the data in each table in a matrix.

1.

Polyhedra			
	Faces	Vertices	Edges
Tetrahedron	4	4	6
Cube	6	8	12
Octahedron	8	6	12
Icosahedron	20	12	30

2.

Tourism in Burma, 1998–99	
	Number of Visitors
From Europe	52,790
From N. America	13,041
From Asia	126,441
From Africa	354

3.

Batting Statistics		
	At Bats	Hits
Johnson	69	22
Crabtree	108	31
Aguirre	47	13

4.

Video Inventory			
	Comedy	Drama	Horror
New Releases	16	19	8
Weekly Rentals	57	62	39

Give the dimensions of each matrix.

5. $\begin{bmatrix} -6 \end{bmatrix}$

6. $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

7. $\begin{bmatrix} 9 & 11 & 7 & 6 \\ 2 & -3 & 4 & 6 \end{bmatrix}$

8. $\begin{bmatrix} 0 \\ 15 \\ 22 \end{bmatrix}$

9. $\begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$

10. Write About It The matrices show the revenue and expenses, in millions of dollars, generated by the three different products that a company sells during two months of the year. Explain how to find the company's total profit for each of the products for those two months. (*Hint:* Profit is revenue minus expenses.)

$$\begin{array}{c} \textbf{Revenue} \\ \begin{array}{ccc} \text{A} & \text{B} & \text{C} \end{array} \\ \begin{array}{c} \text{Month 1} \\ \text{Month 2} \end{array} \begin{bmatrix} 2.3 & 0.7 & 3.0 \\ 2.1 & 1.1 & 2.8 \end{bmatrix} \end{array} \qquad \begin{array}{c} \textbf{Expenses} \\ \begin{array}{ccc} \text{A} & \text{B} & \text{C} \end{array} \\ \begin{array}{c} \text{Month 1} \\ \text{Month 2} \end{array} \begin{bmatrix} 1.8 & 1.6 & 2.1 \\ 1.9 & 1.2 & 2.0 \end{bmatrix} \end{array}$$

Add or subtract.

11. $\begin{bmatrix} -1 & 4 \\ -3 & 6 \end{bmatrix} + \begin{bmatrix} 4 & 6 \\ 2 & -4 \end{bmatrix}$

12. $\begin{bmatrix} 5 & 7 & 3 \\ -4 & 6 & -2 \end{bmatrix} - \begin{bmatrix} 9 & 19 & 6 \\ -3 & 7 & 2 \end{bmatrix}$

13. $\begin{bmatrix} 1.4 \\ -0.6 \end{bmatrix} - \begin{bmatrix} 2.8 \\ -1.3 \end{bmatrix}$

14. $\begin{bmatrix} \frac{3}{5} & \frac{1}{3} & \frac{4}{5} \end{bmatrix} - \begin{bmatrix} \frac{1}{5} & \frac{5}{6} & \frac{3}{10} \end{bmatrix}$

15. Hiking The matrix shows the number of miles hiked on each day of a three-day hiking trip.

    a. Use scalar multiplication to convert the distances to kilometers. Write your answer as a matrix. (*Hint:* A mile is about 1.6 kilometers.)

    b. What is the total length of the hiking trip in kilometers?

$$\begin{array}{c} \text{Miles} \\ \begin{array}{c} \text{Day 1} \\ \text{Day 2} \\ \text{Day 3} \end{array} \begin{bmatrix} 10.2 \\ 18.6 \\ 4.3 \end{bmatrix} \end{array}$$

Multiply.

16. $6 \begin{bmatrix} 3 & 2 \\ -2 & -9 \end{bmatrix}$

17. $-4 \begin{bmatrix} 1 & 0 & -4 \\ 3 & -3 & 5 \\ 0 & 6 & -1 \end{bmatrix}$

18. $2.7 \begin{bmatrix} 1.9 & 2.7 \\ -0.8 & -3.6 \\ 1.4 & 4.2 \end{bmatrix}$

19. $-0.6 \begin{bmatrix} 8.2 & 10 & -2.8 \end{bmatrix}$

20. $\dfrac{2}{3} \begin{bmatrix} \frac{1}{4} & -6 \\ -4 & \frac{3}{2} \end{bmatrix}$

21. $-2\dfrac{1}{2} \begin{bmatrix} -5 & 3 & \frac{4}{5} \\ \frac{1}{5} & -4 & 2 \end{bmatrix}$

22. Entertainment Two teams are competing in a scavenger hunt. Each item on the list is worth 1, 5, or 10 points. The matrix shows the number of items of each point value that was found by the two teams.

	Red team	Blue team
1-point	22	7
5-point	13	18
10-point	4	6

  a. How many items did the red team find?

  b. How many total points were scored by the red team?

  c. How many items did the blue team find?

  d. How many total points were scored by the blue team?

  e. What is the total number of 5-point items found by the two teams?

  f. How many more 1-point items did the red team find than the blue team?

23. Technology Computer animation programs store data in matrices. The data may describe how a figure moves across the screen. Matrix A represents the coordinates of the vertices of a polygon. Matrices E and F represent translations of the polygon.

Matrix *A*		Matrix *E*		Matrix *F*	
x	*y*	*x*	*y*	*x*	*y*
0	0	0	3	−2	0
4	1	0	3	−2	0
2	5	0	3	−2	0
−2	3	0	3	−2	0

  a. Graph the ordered pairs given by matrix A and connect them in the order they are listed. Connect the last point to the first point.

  b. Add matrices A and E and graph the new ordered pairs. Connect the points as in part **a** and describe how the graph is different from part **a.**

  c. Subtract matrix F from matrix A and graph the new ordered pairs. Connect the points and describe how the graph is different from part **a.**

  d. Multiply matrix A by 2 and graph the new ordered pairs. Connect the points and describe how the graph is different from part **a.**

  e. Add matrices A and F, and then multiply the result by $\frac{1}{2}$. Graph the new ordered pairs and connect the points. Describe how the graph is different from part **a.**

  f. Challenge Use the following information to write matrix G:
Matrices A and G are added and the new ordered pairs are graphed. The points are connected as in part **a.** The graph looks like the graph from part **a** has been shifted down 2 units and to the right 3 units.

Vocabulary

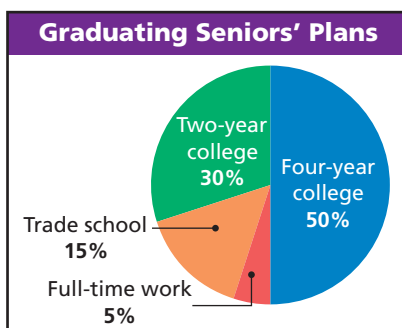Complete the sentences below with vocabulary words from the list above.

1. A(n) ___?___ is one possible result of an experiment.

2. The ___?___ is the difference between the upper and lower quartiles.

3. Two events are ___?___ if the occurrence of one event does not affect the probability of the other.

## 10-1 Organizing and Displaying Data (pp. 678–686)

TEKS A.1.D, A.2.D

EXAMPLE

The circle graph shows the post-graduation plans for a high school's 500 graduating seniors.

Graduating Seniors' Plans

Two-year college **30%**
Four-year college **50%**
Trade school **15%**
Full-time work **5%**

■ How many seniors plan to attend a four-year college?
50% of 500   *Find the percentage.*
$0.50 \times 500 = 250$
250 students plan to attend a four-year college.

EXERCISES

Use the double-bar graph for Exercises 4 and 5.

Intramural Soccer Participation

Students vs Year (2002, 2003, 2004, 2005)
Legend: Boys, Girls
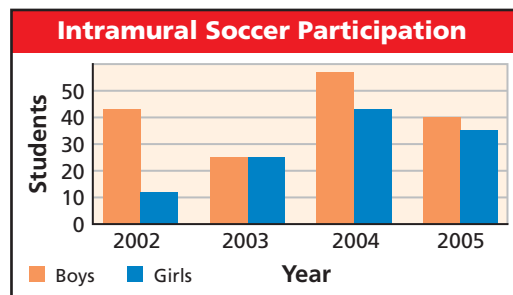
4. In which year did the same number of boys and girls participate?

5. How many more boys than girls participated during 2004?

10-2 Frequency and Histograms (pp. 687–693)

EXAMPLE

- **The lives (in hours) of each light bulb in a test are given below. Use the data to make a frequency table and a histogram.**

    22, 28, 25, 21, 19, 21, 25,
    21, 22, 18, 20, 29, 25, 26

    Identify the least and greatest values. Divide the data into equal intervals.

Light Bulb Life (h)	
Life	**Frequency**
18–20	3
21–23	5
24–26	4
27–29	2

Draw a bar for the number of hours in each interval. The bars touch, but do not overlap.
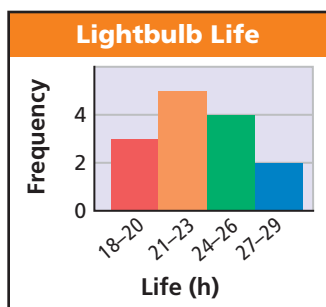
EXERCISES

6. The weights of packages shipped by an online retailer are given below. Use the data to make a stem-and-leaf plot.

Package Weight (lb)
14 9 22 24 7 1 19 22 28 18 12

7. The numbers of people who attended two different plays are shown in the table below. Make a back-to-back stem-and-leaf plot.

Play Attendance	
Comedy Camp	**Days and Days**
104 62 83 102	103 105 80 135
104 120 81	109 128 82
126 122	132 139

8. The capacities of the gas tanks on several new vehicles are shown below. Use the data to make a frequency table with intervals.

Gas Tank Capacity (gal)
15 12 12 15 18 26 25 12 15 18 11
10 12 16 15 16 18 25 21 18 20 21

9. Use your frequency table from Exercise 9 to make a histogram.

10-3 Data Distributions (pp. 694–699)

EXAMPLE

- **Consider the following ages of the winners of an art contest: 13, 15, 14, 18, 12, 10, 11, 13. Find the mean and mode of the data.**

    mean:

    $$\frac{10 + 11 + 12 + 13 + 13 + 14 + 15 + 18}{8}$$

    $$= \frac{106}{8}$$

    $$= 13.25$$

    mode: 13   *13 occurs most often.*

EXERCISES

Find the mean, median, mode, and range of each data set.

10. Years of playing experience of the members of a musical ensemble:

    5, 14, 25, 7, 8, 10, 12, 33, 12

Herman has five pairs of cowboy boots. The prices were $120, $137, $120, $145, and $482.
mean: $200.80   median: $137   mode: $120

11. Which value best describes the price Herman paid? Explain.

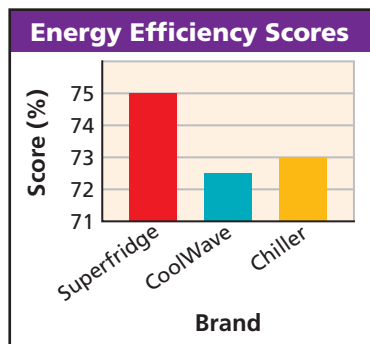**Use the data to make a box-and-whisker plot.**

12. 25, 28, 2, 24, 28, 21, 18, 29, 31, 12, 6, 19, 27, 3

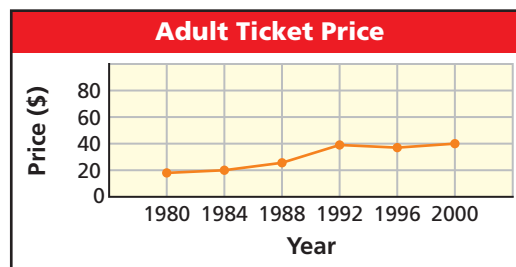## 10-4 Misleading Graphs and Statistics (pp. 702–709)

TEKS A.1.D

EXAMPLE

■ Explain why the graph is misleading.

Energy Efficiency Scores

The vertical axis begins at 71. This exaggerates the difference in the heights of the bars.

EXERCISES

The graph shows the cost of admission to an amusement park over 20 years.

Adult Ticket Price

13. Explain why the graph is misleading.

14. What might someone believe because of the graph?

10-5 Experimental Probability (pp. 713–718)

EXAMPLES

The manager of a photo processing lab inspects 500 photos and finds that 4 have flaws.

■ What is the experimental probability that a photo is flawed?

$$\frac{\text{number of times the event occurs}}{\text{number of trials}} = \frac{4}{500} = 0.8\%$$

■ In one month, the lab processes 13,000 photos. Predict the number that are likely to be flawed.

$0.008 \cdot 13,000 = 104$ *Find 0.8% of 13,000*

104 photos are likely to be flawed.

EXERCISES

A manufacturer inspects 800 batteries and finds that 796 have no defects.

15. What is the experimental probability that a battery chosen at random has no defects?

16. There are 25,000 batteries in storage. How many batteries are likely to have no defects?

17. Another storage area holds 50,000 batteries. How many batteries are likely to have a defect?

10-6 Theoretical Probability (pp. 720–725)

EXAMPLE

■ A jar contains red, green, brown, and blue marbles. The probability of choosing red is 0.30, of choosing green is 0.20, and of choosing brown is 0.25. Find the probability of choosing blue.

$$P(\text{blue}) + P(\text{not blue}) = 1$$
$$P(\text{blue}) + P(\text{red, green, or brown}) = 1$$
$$P(\text{blue}) + (0.30 + 0.20 + 0.25) = 1$$
$$P(\text{blue}) + 0.75 = 1$$
$$P(\text{blue}) = 0.25$$

EXERCISES

Find the theoretical probability.

18. Rolling a number less than 4 on a standard number cube

19. Randomly selecting a month that starts with "J" from all month names

20. Randomly selecting a vowel from the letters in EQUATION

10-7 Independent and Dependent Events (pp. 726–733)

EXAMPLES

A hardware store shelf holds 12 cans of red paint, 4 cans of yellow paint, and 6 cans of black paint.

■ Syd selects one can at random and replaces it. Then she selects another can at random. What is the probability that Syd selects a red can and then a yellow can?

$P(\text{red, yellow}) = P(\text{red}) \cdot P(\text{yellow})$

$\qquad = \dfrac{12}{22} \cdot \dfrac{4}{22}$ *Independent events*

$\qquad = \dfrac{48}{484} = \dfrac{12}{121}$

■ Gene selects one can at random and then selects another can at random from the remaining cans. What is the probability that Gene selects two cans of black paint?

$P(\text{black, black}) = P(\text{black}) \cdot P(\text{black after black})$

$\qquad = \dfrac{6}{22} \cdot \dfrac{5}{21}$ *Dependent events*

$\qquad = \dfrac{30}{462} = \dfrac{5}{77}$

EXERCISES

Tell whether each set of events is independent or dependent. Explain your answers.

21. A computer generates a random number and then generates another random number.

22. You roll two number cubes. One is a 6 and the other is a 1.

23. Two audience members are called to the stage.

A lottery machine contains different-colored balls. There are 64 green, 128 yellow, 1 golden, and 3 silver balls. Find the probability of each event.

24. A yellow ball is drawn and set aside. Then a green ball is drawn.

25. A golden ball is drawn and set aside. Then another golden ball is drawn.

26. A green ball is drawn and replaced. Then another green ball is drawn.

10-8 Combinations and Permutations (pp. 736–743)

EXAMPLES

A sporting goods store carries sweatshirts for 8 local high school football teams.

■ How many different packages of 4 different high school sweatshirts are possible?
Use a combination. The order does not matter.

$_8C_4 = \dfrac{8!}{4!(8-4)!} = \dfrac{8!}{4!(4)!}$

$\qquad = \dfrac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{(4 \cdot 3 \cdot 2 \cdot 1)(4 \cdot 3 \cdot 2 \cdot 1)} = \dfrac{1680}{24} = 70$

Seventy different packages are possible.

■ Three different high school sweatshirts will be hung in a row. How many displays are possible?
Use a permutation. The order matters.

$_8P_3 = \dfrac{8!}{(8-3)!} = \dfrac{8!}{5!} =$

$\dfrac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 336$

There are 336 different possible displays.

EXERCISES

27. A catering hall offers 4 different plates, 3 different silverware patterns, and 5 different types of glassware. How many place settings of one plate, one silverware pattern, and one type of glassware are possible?

Tell whether each situation involves combinations or permutations. Then give the number of possible outcomes.

28. Shelly is making up a 7-digit phone number to use in a play. She can choose any digit from 0–9 but does not want to repeat a number. How many different phone numbers are possible?

29. A restaurant offers 12 different appetizers. How many ways can a group of friends share 3 different appetizers?

30. A group of 15 friends is at an amusement park. In how many ways can a group be chosen to ride in a four-person gondola?

1. The table shows the population of Oakville. Use the data to make a graph. Explain why you chose that type of graph.

2. Which ten-year period saw the greatest change in population?

3. Describe the trend in Oakville's population.

Population of Oakville	
Year	Population
1970	20,851
1980	14,229
1990	11,198
2000	9,579

The high temperatures in degrees Fahrenheit for two weeks are given:
64, 66, 63, 58, 59, 55, 51, 54, 61, 62, 68, 70, 63, 63.

4. Use the data to make a stem-and-leaf plot.

5. Use the data to make a frequency table with intervals.

6. Use your frequency table from Problem 5 to make a histogram.

The lengths of statements during a town council meeting are given.

7. Find the mean, median, mode, and range of the data.

8. Use the data to make a box-and-whisker plot.

Length of Statement (min)						
6	25	12	14	2	13	38
22	21	14	3	8	5	17

A manufacturer inspects 500 watches and finds that 498 have no defects.

9. What is the experimental probability that a watch chosen at random has no defects?

10. There are 30,000 watches in a warehouse. Predict the number of watches that are likely to have no defects.

The graph shows how the money raised by a charity is spent.

11. Explain why the graph is misleading.

12. What might someone believe because of the graph?

13. Who might want to use this graph?

14. An experiment consists of pulling one card out of a bag that has 12 cards, each with a different month of the year printed on it. What is the probability that the month begins with *A*?

Charity Spending

Administrative costs **4%**

Postage costs **1%**

Charitable causes **15%**
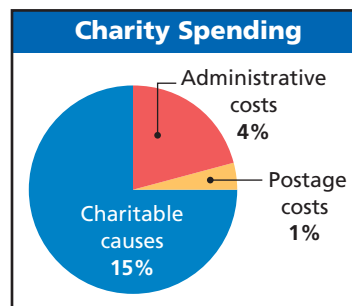
15. The odds of spinning red on a spinner are 2:7. What is the probability of not spinning red?

16. A bag has 14 red marbles and 10 white marbles. Rosa randomly picks two marbles from the bag, one at a time. What is the probability that Rosa picks two white marbles?

Tell whether each situation involves combinations or permutations. Then give the number of possible outcomes.

17. Sara is ordering a deli platter for a party. The deli offers 9 kinds of meats. Sara must select 4 kinds for the platter. How many different platters can Sara choose?

18. Armando has 12 antique books. He will select 3 books and put one up for auction each day for three days. How many different ways can he do this?

COLLEGE ENTRANCE EXAM PRACTICE

FOCUS ON SAT MATHEMATICS SUBJECT TEST

There are two levels of SAT Subject Tests: Math Level I and Math Level II. Each test has 50 multiple-choice questions. The content of each test is very different. Getting a high score on one test does not mean you will get a high score on the other test.

HOT TIP! You can write all over the test book to sketch figures, do scratch work, or cross out incorrect answers to help you eliminate choices. Remember to mark your final answer on the answer sheet because the test book is not used for scoring.

You may want to time yourself as you take this practice test. It should take you about 6 minutes to complete.

---

1. Which of the following represents the number of ways 6 committee members can be selected from a group of 25 teachers?

(A) $_6C_{25}$

(B) $_{19}C_6$

(C) $_{25}C_6$

(D) $_6P_{25}$

(E) $_{25}P_6$

---

2. Your friend randomly selects 2 movies from your collection to borrow. Your collection includes 9 comedies, 5 dramas, 3 mysteries, 1 musical, and 2 horror movies. What is the probability that both movies are comedies?

(A) $\dfrac{1}{36}$

(B) $\dfrac{18}{95}$

(C) $\dfrac{81}{400}$

(D) $\dfrac{2}{9}$

(E) $\dfrac{331}{380}$

---

3. A student's test scores are 88, 87, 82, 95, and 89. If the student needs an average test score of 90 to obtain a grade of A, what must the score be on the last test of the semester?

(A) 88

(B) 88.2

(C) 91

(D) 99

(E) 100

---

4. What is the probability of not getting a 2 on the spinner?
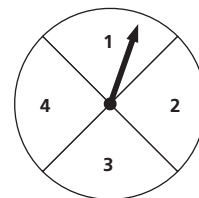
(A) 0.10

(B) 0.25

(C) 0.50

(D) 0.75

(E) 1

---

5. What is the value of $\dfrac{(n+1)!}{(n-1)!}$?

(A) $n^2 + n$

(B) $2n + 1$

(C) $n!$

(D) $n^2 - 1$

(E) -1

Any Question Type: Spatial Reasoning

Some test questions include a three-dimensional figure. You must use spatial reasoning to correctly interpret the figure.

EXAMPLE 1

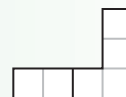**Which of the following shows the top view of the solid figure?**
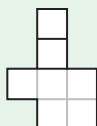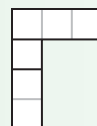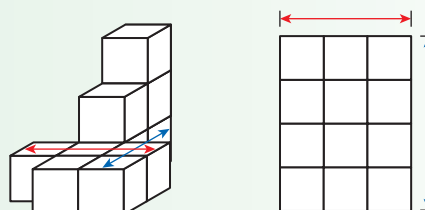
A   B   C   D

Notice that the isometric drawing shows three sides of the figure: the top, front, and right side. Use the isometric drawing to make a foundation plan of the figure.

Count the number of cubes at the widest part of the figure and at the longest part of the figure.

The widest part is 3 cubes across.
The longest part is 4 cubes deep.

Draw a 3-by-4 grid.

Each cube on the bottom level of the figure corresponds to a square on the grid.
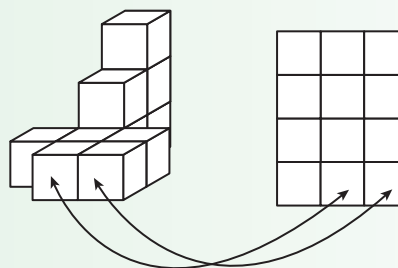
Start at the front of the figure. Shade in each square of the grid that corresponds to a cube on the bottom level of the figure.
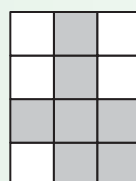
Using the foundation plan, you can see that choice B shows the top view of the solid.

Read each test item and answer the questions that follow.

Item A
Multiple Choice The drawing shows a foundation plan of a solid figure made of stacked cubes. The numbers in the squares identify the number of cubes in each stack. Which three-dimensional view represents this solid figure?
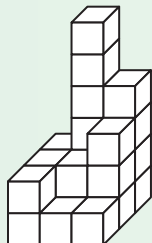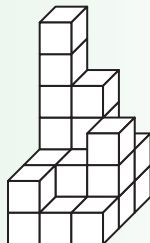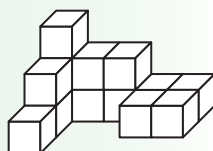
2	4	6
2	2	3
2	1	1

F

H

G

J
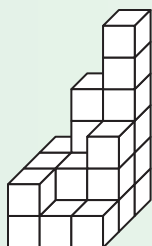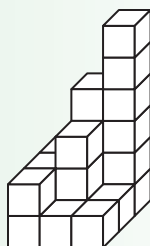
1. How many cubes high is the highest stack? Explain how you know.

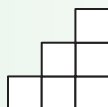2. A student answered choice G. Explain whether you agree with the student's choice.

Item B
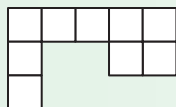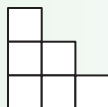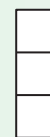**Multiple Choice** Which is the top view of the solid figure?

A

B

C

D
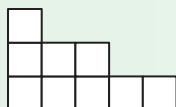
3. If you look down on the figure, how many cubes do you see?

4. Create a foundation plan for the figure.

5. If you look at the figure from the front, how tall is the highest stack of cubes and how short is the shortest stack of cubes?
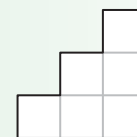
Item C
Multiple Choice Which solid is represented by these top, side, and front views?
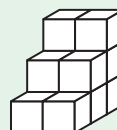
Top   Front   Right side

A

B

C

D

6. How many cubes can you see if you are looking down on the figure? How do you know?

7. How many cubes can you see if you are looking at the front of the figure? How do you know?

8. How many cubes high is the highest stack of the figure? Explain how you know.

9. Using your answers from Problems 1–3, what is the correct answer choice?

CHAPTER
10

TAKS PREP

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TAKS Practice Online
KEYWORD: MA7 TestPrep

| TAKS | Grade 8 Obj. 1, 2, 5, 6 |
| | Grades 9–11 Obj. 1–4, 9, 10 |

CUMULATIVE ASSESSMENT, CHAPTERS 1–10

Multiple Choice

1. Josephine has 9 cousins. Her friend Summer has *s* more cousins. Which expression can be used to show the number of cousins Summer has?

(A) $9 > s$

(B) $9s$

(C) $9 + s$

(D) $9 - s$

2. A negative number raised to an exponent is positive. Which of the following is NOT true?

(F) The number could be even.

(G) The number could be odd.

(H) The exponent could be even.

(J) The exponent could be odd.

3. A computer game normally sells for $40. Niko bought it when it went on sale for 25% off. The sales tax was 6%. How much did Niko pay for the game?

(A) $10.60

(B) $12.40

(C) $31.80

(D) $32.40

4. The graph shows the profit made at a family yard sale that lasted 8 hours. Between which hours did the profit increase at the slowest rate?

Yard Sale

(F) 0–1

(G) 1–2

(H) 5–6

(J) 7–8
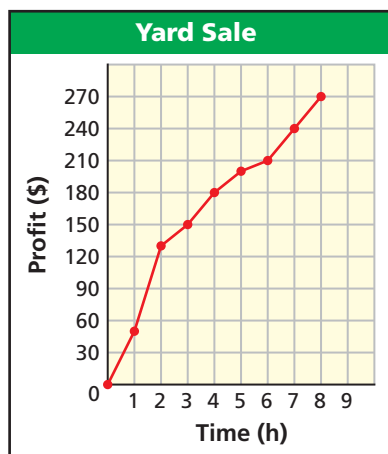
5. Which of the following is one of the five values needed to make a box-and-whisker plot?

(A) Mean

(B) Median

(C) Mode

(D) Average

6. Which value is a solution of $2(1 - x) < 12$?

(F) −7

(G) −6

(H) −5

(J) −4

7. What is the median of the data presented in this stem-and-leaf plot?

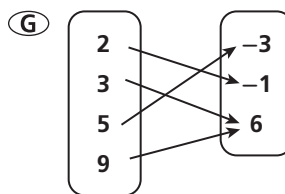| Stem	Leaves
6	0 2 2 2 5 8
7	3 4 4 6 7
8	2 5 8 9

Key: 8 | 2 means 0.82

(A) 0.62

(B) 0.74

(C) 6.2

(D) 7.4

8. Which of the following does NOT represent a function?

(F) $\{(-4, 3), (-2, 1), (-2, -1), (-4, -3)\}$
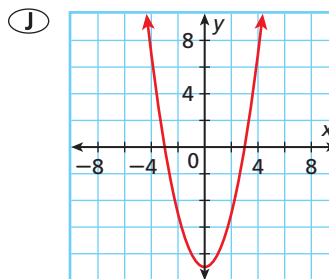
(G)

(H)

x	−4	−3	−2	−1	0	1
y	4	3	2	1	0	−1

(J)

9. Which of these equations describes a line parallel to $x + 2y = 6$?

   (A) $y = \frac{1}{2}x$ (C) $y = -2x + 6$

   (B) $y = -\frac{1}{2}x + 4$ (D) $y = 2x + 3$

10. What is the probability that the spinner will NOT land on yellow?
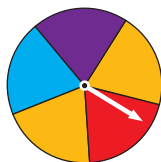
    (F) 20%

    (G) 40%

    (H) 50%

    (J) 60%

11. A CD-ROM is storing 4.2×10^8 bytes of information. What is this number in standard form?

    (A) 0.0000000042

    (B) 0.00000042

    (C) 420,000,000

    (D) 4,200,000,000

12. Which is a factor of $4x^2 + 24x + 36$?

    (F) $(x + 12)$

    (G) $(2x + 6)$

    (H) $(2x + 18)$

    (J) $(4x + 18)$

Gridded Response

13. Yvette is choosing between two stores to get her family pictures taken. Say Cheese charges a $20 sitting fee and $14 per sheet of pictures. Picture Me charges a $12 sitting fee and $18 per sheet of pictures. For how many sheets will the total cost be the same at both stores?

14. Brian plays basketball. In the past, he has made 4 out of every 5 free throws. What is the probability, as a decimal, that Brian will make the next two free throws?

15. A password for a security system must consist of a letter followed by two digits. How many possible passwords are there?

Short Response

16. An experiment consists of tossing 3 coins at the same time.

    a. Identify the sample space.

    b. What is the probability of tossing 2 tails and 1 head?

    c. Which is more likely to occur: tossing exactly 1 tail or tossing at least 2 heads? Explain.

17. If 8% of a number is 4, what is 12% of the number? Explain your reasoning.

18. Nat surveyed 30 twelfth-grade students and found that 27 of them were in favor of allowing seniors to leave campus for lunch. As a result, Nat reported that "90% of the student body supports off-campus lunch privileges for seniors." Was Nat's statistic misleading? Explain.

19. A service technician is scheduling house calls to make repairs. He has 9 time slots in his work day. He has 7 house calls to make. How many different schedules of his day are possible? Show and explain your work.

20. For her scrapbook page, Denise has 5 different background colors, 4 different framing styles, and 2 different accents. How many possible scrapbook layouts can Denise create with the available materials? Show and explain your work.

Extended Response

21. The ages of the first 15 people to enter a community center for bingo one evening are 77, 85, 76, 73, 69, 70, 84, 82, 76, 74, 89, 83, 68, 91, and 88.

    a. Find the mean, median, mode, and range of the data.

    b. Create a box-and-whisker plot of the data.

    c. The sixteenth person to enter the center is 39 years old. Calculate the mean, median, mode, and range of the data with the 16 people.

    d. Create a box-and-whisker plot with the 16 people.

    e. When the sixteenth person was added, which measure of central tendency changed the most? Which measure changed the least? Explain.

    f. How did the addition of the sixteenth person affect the box-and-whisker plot?

Problem Solving on Location
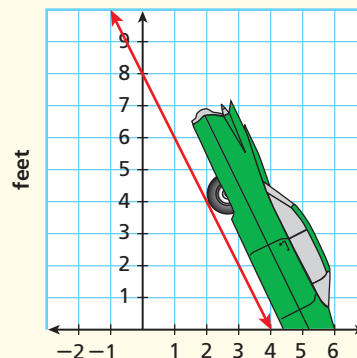
TEXAS

Amarillo

The Mother Road

Route 66 was one of the first national highways. It stretched more than 2000 miles from Chicago, Illinois, to Santa Monica, California, and 178 miles crossed through the Texas panhandle. It was gradually replaced by interstate highways until it was decommissioned in 1985.

HISTORIC
TEXAS
U S
66
ROUTE

Choose one or more strategies to solve each problem.

1. Route 66 was very popular with tourists in the 1950s when gas cost about 25¢/gal. A modern hybrid car can get around 50 mi/gal, but 2005 gas prices are close to $2/gal. About how much money would the 178-mile trip along Texas' Route 66 cost by today's standards? In 1950, the trip might have cost $2.97. Estimate how many miles per gallon the car was consuming.

For 2, use the graph.

2. It has been suggested that the cars at Cadillac Ranch were positioned to have a slope equal to that of the sides of the Great Pyramid in Egypt. The equation of a line drawn on the side of the pyramid is $y + 1.27x = 481$. Write the equation of the line in the graph, and then make a statement comparing the slope of the car with that of one side of the Great Pyramid.

feet

For 3, use the map.

3. The town of Adrian marks the halfway point of Route 66. The distance from Texola, near the Oklahoma-Texas border, to this point is about 156 miles. A road map with a scale of 1-inch-to-20-miles shows the distance from Texola to McLean to be 1.75 inches. How many miles is it from McLean to Adrian?

GAS

MIDPOINT
CAFE

NEW MEXICO
Glenrio
Adrian
Wildorado
Amarillo
Cadillac Ranch
Conway
Groom
McLean
Shamrock
Texola
OKLAHOMA

TEXAS
U S
66

BIG TEXAN
STEAK RANCH

TEXAS

Palo Duro Canyon

DROP
INN
CAFE

The Great Texas Birding Classic

Texas has a wide variety of bird species and serves as a "superhighway" for migrating birds. The Great Texas Birding Classic is a race to see, hear, and record as many bird species as possible.

1. Birders rely on range maps to help them find and identify birds. What is the probability that a birder might see a Northern Caracara in Canada? Which is greater, the probability of seeing a Caracara on the Texas coast, or seeing one in the Panhandle?

2. One event at the Birding Classic tests how well birders can identify birds by their songs. Assume that most songbirds have 10 distinct songs and that an experienced birder can recognize 200 songs. What is the probability that an experienced birder will recognize a particular song, given that there are approximately 500 bird species in Texas?

3. The quality of bird range data is dependent upon how many sighting reports are submitted. Make a graph that explains whether the Caracara population is increasing or decreasing.

Year	Reports Submitted	Caracara Sightings
2001	216	1
2002	847	5
2003	2341	32
2004	3034	61

120

Crested Caracara
Caracara cheriway

Height 23 in. (58 cm) • Wingspan 50 in. (127 cm) • Weight 2.2 lb (990 g)

black cap

McKinney Falls
State Park
8-9-05
soaring
@ dusk

wide wings

long legs

white wingtips

Year-round range
Rare occurrence

Long neck, distinctive long le
Blackish-brown coverts, white
throat and neck, bare facial ski
changes color from red-orange
to yellow. **Juvenile** is more
brown, underparts edged in bu
adult shows more barring und
tail. Steady, ravenlike wingbea
Call is a low, croaking *gyrrrk*
single track. **Range:** Common
in Texas range area, common
through Mexico and northern
South America. Records from
beyond typical range may be c
escaped birds.